Diagonalizability

review $\det(\underset{n\times n}{A} - \lambda I)$ characteristic polynomial (degree n)

$\overset{\text{factor}}{\underset{(*)}{=}} (\lambda_1 - \lambda)(\lambda_2 - \lambda) \cdots (\lambda_n - \lambda)$ λ_i eigenvalues

THM $\det(\underset{n\times n}{A}) = \lambda_1 \lambda_2 \cdots \lambda_n$ product of all eigenvalues

Pf Set $\lambda = 0$ in $(*)$.

EG $\underset{2\times 2}{A}$, $\det(A) = -8$, $\lambda_1 = 4$

$\Longrightarrow \lambda_2 = \frac{\det(A)}{\lambda_1} = -\frac{8}{4} = -2$

THM $\underset{n\times n}{A}$ diagonalizable

$\Longleftrightarrow$ A has n linearly independent eigenvectors

$A = PDP^{-1}$

$\Longleftrightarrow AP = PD$

matrix version of $Av = \lambda v$

$\Longleftrightarrow$ for each eigenvalue λ of multiplicity k,

$\dim \underbrace{\text{null}(A - \lambda I)}_{\lambda\text{-eigenspace}} = k$

could be $1, 2, \ldots, k$

EG $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$ $\det(A - \lambda I) = \begin{vmatrix} -\lambda & 1 \\ 0 & -\lambda \end{vmatrix} = \lambda^2$

eigenvalue $\lambda = 0$ multiplicity 2

0-eigenspace $= \text{null}\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ has $\dim = 1$

basis : $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

$\Longrightarrow$ A not diagonalizable

EG $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ $\det(A - \lambda I) = \begin{vmatrix} -\lambda & -1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 + 1$

$\overline{a+bi} = a - bi$

eigenvalues $\lambda = \pm i$

i-eigenspace ($-i$) $= \text{null}\begin{pmatrix} -i & -1 \\ 1 & -i \end{pmatrix}$ basis : $\begin{bmatrix} i \\ 1 \end{bmatrix}$ $\begin{bmatrix} -i \\ 1 \end{bmatrix}$

$\Longrightarrow A = PDP^{-1}$ $D = \begin{bmatrix} i & \\ & -i \end{bmatrix}$ $P = \begin{bmatrix} i & -i \\ 1 & 1 \end{bmatrix}$