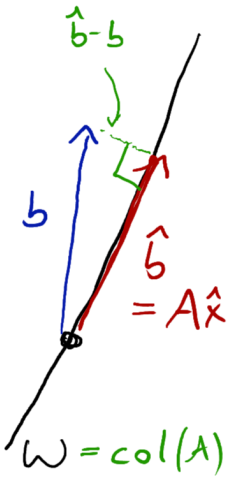
# Orthogonal projections




**DEF** orthogonal projection $\hat{b}$ of $b$ onto a subspace $W$

= the vector in $W$ closest to $b$

$= \text{col}(A)$

$\hat{b} = A\hat{x}$ where $\hat{x}$ is the least squares solution to $Ax = b$

[i.e. $A^TA\hat{x} = A^Tb$]

**EG** Orthogonal projection of $\begin{bmatrix} 2 \\ 0 \\ 11 \end{bmatrix} = b$

onto $\text{span}\left\{\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}\right\} = \text{col}(A)$ $\quad A = \begin{bmatrix} 4 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$

projection $\hat{b}$

$= A\hat{x}$

$= 1 \cdot \begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix} + 2 \cdot \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

$= \begin{bmatrix} 4 \\ 4 \\ 3 \end{bmatrix}$

$\hat{x}$ least squares sol. of $Ax = b$

$A^TA\hat{x} = A^Tb$ normal equations

$\begin{bmatrix} 17 & 1 \\ 1 & 5 \end{bmatrix}\hat{x} = \begin{bmatrix} 19 \\ 11 \end{bmatrix}$

$\Rightarrow \hat{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

check $\hat{b} - b$ orthogonal to $\text{col}(A)$ [i.e. $\begin{bmatrix} 4 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$]

error $= \begin{bmatrix} 2 \\ 4 \\ -8 \end{bmatrix}$