Consistency

warmup

$$\begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1 \begin{bmatrix} 1 \\ 3 \\ 0 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$$

A x — linear combination of cols of A

$Ax = b$ **consistent** i.e. there is a solution x

$\iff$ b is in $\text{col}(A)$

FTLA

$\iff$ b orthogonal to $\text{null}(A^T)$

FTLA

$\text{col}(A)^\perp = \text{null}(A^T)$

EG

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 1 \\ 0 & 5 \end{bmatrix}$$

For which b is $Ax = b$ consistent?

old approach

$$\left[\begin{array}{cc|c} 1 & 2 & b_1 \\ \textcircled{3} & 1 & b_2 \\ 0 & 5 & b_3 \end{array}\right] \overset{R_2 - 3R_1 \Rightarrow R_2}{\rightsquigarrow} \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 0 & -5 & -3b_1 + b_2 \\ 0 & \textcircled{5} & b_3 \end{array}\right] \overset{R_3 + R_2 \Rightarrow R_3}{\rightsquigarrow} \left[\begin{array}{cc|c} 1 & 2 & b_1 \\ 0 & -5 & -3b_1 + b_2 \\ 0 & 0 & -3b_1 + b_2 + b_3 \end{array}\right]$$

must be zero!

$Ax = b$ consistent

$\iff -3b_1 + b_2 + b_3 = 0$

new approach

basis for $\text{null}(A^T)$: $\begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix}$

$$\underset{A^T}{\begin{bmatrix} 1 & 3 & 0 \\ 2 & 1 & 5 \end{bmatrix}} \overset{\text{RREF}}{\rightsquigarrow} \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -1 \end{bmatrix}$$

$Ax = b$ consistent

$\iff$ b is orthogonal to $\text{null}(A^T)$

$\iff b \cdot \begin{bmatrix} -3 \\ 1 \\ 1 \end{bmatrix} = 0$ as above!