Review : null spaces

- vectors — can be added + scaled
- span $\{v_1, \dots, v_n\}$ = set of all linear combinations $\lambda_1 v_1 + \dots + \lambda_n v_n$
- vector spaces "=" spans
- basis of V — $v_1, \dots, v_n$ from V form a basis of V
  $\iff$ every vector in V can be expressed as a unique linear combination of $v_1, \dots, v_n$
  $v_1, \dots, v_n$ span V
  $v_1, \dots, v_n$ linearly independent
- dimension of V = number of vectors in (any) basis of V

EG $\mathbb{R}^3$ = space of all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ with x, y, z in $\mathbb{R}$

$x \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + y \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + z \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

standard basis : $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$

dim = 3

EG if $Ax = 0$ and $Ay = 0$ then $A(x+y) = 0$

null space $\quad$ null(A) = space of all x such that $Ax = 0$

EG

$$A = \begin{bmatrix} 1 & 1 & 2 & 1 \\ 3 & 2 & 7 & 2 \\ -2 & 6 & -3 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 0 & 5/3 \\ 0 & 1 & 0 & 4/9 \\ 0 & 0 & 1 & -5/9 \end{bmatrix} = R$$

pivots

x_4 free

$$3x_1 + 2x_2 + 7x_3 + 2x_4 = 0$$

important $\quad$ null(A) = null(R)

$$x_2 + \frac{4}{9} x_4 = 0$$

$Rx = 0$ general sol $\begin{bmatrix} -5/3\, x_4 \\ -4/9\, x_4 \\ 5/9\, x_4 \\ x_4 \end{bmatrix}$

$$x_1 = -\frac{5}{3} x_4$$
$$x_2 = -\frac{4}{9} x_4$$
$$x_3 = \frac{5}{9} x_4$$

$$\implies \text{null}(A) = \text{span}\left\{ \begin{bmatrix} -5/3 \\ -4/9 \\ 5/9 \\ 1 \end{bmatrix} \right\}$$

is a basis for null(A)

EG

$$A = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 0 & 2 \\ 1 & 0 & 1 \end{bmatrix} \xrightarrow{\text{RREF}} \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

pivot

x_2, x_3 free

$x_1 = -x_3$

$Ax = 0$ general sol $\underbrace{\begin{bmatrix} -x_3 \\ x_2 \\ x_3 \end{bmatrix}}$

$$= x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

$$\text{null}(A) = \text{span}\left\{ \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

basis : $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$

$\dim(\text{null}(A)) = 2$
\# of free variables