Crash course in linear algebra, I

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

row vector

column vector

2×3 matrix

vector operations:

- addition
- scaling

matrix multiplication

$$\begin{bmatrix} a & b & c \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = ax + by + cz$$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \\ 2 & -2 \end{bmatrix} = \begin{bmatrix} 4 & -3 \\ 7 & -5 \end{bmatrix}$$

$1 \cdot 1 + (-1) \cdot (-1) + 1 \cdot 2$

$1 \cdot 0 + (-1) \cdot 1 + 1 \cdot (-2)$

i^{th} row times j^{th} col $\rightsquigarrow$ entry (i,j)

row i, col j

$$\begin{aligned} 2x_1 + 7x_2 &= 3 \\ 3x_1 - x_2 &= 5 \end{aligned} \quad \rightsquigarrow \quad \begin{bmatrix} 2 & 7 \\ 3 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

identity matrix

$$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$AI = A = IA$$

- associative law: $A(BC) = (AB)C = ABC$ (no ambiguity)
- distributive law: $A(B+C) = AB + AC$ (and other way around)
- commutative law does not hold!

$$\begin{bmatrix} 2 & 4 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ 6 & 4 \end{bmatrix}$$

scales rows

scales cols

matrix transpose

$$A^T$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}^T = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$$