Preparing for the Final

MATH 316 — Linear Algebra II
Monday, May 3

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

- Make sure that you have completed all homework.
- Review the practice problems for both midterms (for the material up to Midterm #2).
- The problems below cover the material since Midterm #2.

Problem 2. Consider $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$.

(a) Determine the SVD of A.

(b) Determine the best rank 1 approximation of A.

(c) Determine the pseudoinverse of A.

(d) Find the smallest solution to $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(Then, as a mild check, compare its norm to the obvious solution $\boldsymbol{x} = [\, 1 \;\; 1 \;\; 0 \,]^T$.)

Problem 3.

(a) Determine the SVD of $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$.

(b) Determine the best rank 1 approximation of $A = \begin{bmatrix} 2 & 0 \\ 0 & 2 \\ 1 & 1 \end{bmatrix}$.

(c) Determine the SVD of $A = \begin{bmatrix} 2 & 1 \\ 2 & 1 \end{bmatrix}$.

Problem 4. Find the best approximation of $f(x) = x$ on the interval $[0, 4]$ using a function of the form $y = a + b\sqrt{x}$.

Armin Straub
straub@southalabama.edu

Problem 5.

(a) If A has λ-eigenvalue $\boldsymbol{v}$, then A^3 has [ ].

(b) A is singular if and only if [$\dim \operatorname{null}(A)$].

(c) If $A = \begin{bmatrix} i & 1+2i \\ 3 & 4 \\ 5i & 6-i \end{bmatrix}$, then its conjugate transpose is $A^* =$ [ ].

(d) The norm of the vector $\boldsymbol{v} = \begin{bmatrix} 1-i \\ 2i \end{bmatrix}$ is $\|\boldsymbol{v}\| =$ [ ].

(e) By Euler's identity, $e^{ix} =$ [ ].

(f) What exactly does it mean for a matrix A to have full column rank?

(g) The pseudoinverse of $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & -7 & 0 \end{bmatrix}$ is $A^+ =$ [ ].

(h) If A is invertible then its pseudoinverse is $A^+ =$ [ ].

(i) If A has full column rank then its pseudoinverse is $A^+ =$ [ ].

(j) Suppose the linear system $A\boldsymbol{x} = \boldsymbol{b}$ has infinitely many solutions $\boldsymbol{x}$.

Which of these solutions is produced by $A^+\boldsymbol{b}$? [ ]

(k) Write down the 2×2 rotation matrix by angle θ.

Armin Straub
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