

Midterm #1

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points)

(a) Find the least squares solution to $\begin{bmatrix} 1 & -1 \\ 1 & 0 \\ 1 & 1 \\ 1 & 1 \end{bmatrix} \mathbf{x} = \begin{bmatrix} 2 \\ 4 \\ 2 \\ -1 \end{bmatrix}$.

(b) Determine the least squares line for the data points $(-1, 2), (0, 4), (1, 2), (1, -1)$.

Problem 2. (9 points)

- (a) Using Gram-Schmidt, obtain an orthonormal basis for $W = \text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix} \right\}$.
- (b) Determine the orthogonal projection of $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$ onto W .
- (c) Determine the QR decomposition of the matrix $A = \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}$.
- (d) Determine a basis for the orthogonal complement W^\perp .

Problem 3. (3 points) We want to find values for the parameters a, b, c such that $z = a\sqrt{x} + b + cy^2$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \dots$. Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Problem 4. (3 points) Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$.

- (a) A basis for $\text{null}(A)$ is . A basis for $\text{col}(A)$ is .
- (b) $\dim \text{col}(A) =$, $\dim \text{row}(A) =$, $\dim \text{null}(A) =$, $\dim \text{null}(A^T) =$.

Problem 5. (9 points) Fill in the blanks.

- (a) $\text{null}(A)$ is the orthogonal complement of . $\text{row}(A)$ is the orthogonal complement of .
- (b) If A is a 5×7 matrix with rank 4, then $\dim \text{col}(A) =$ and $\dim \text{null}(A) =$.
- (c) $\hat{\mathbf{x}}$ is a least squares solution of $A\mathbf{x} = \mathbf{b}$ if and only if .
- (d) By definition, a matrix Q is orthogonal if and only if .
- (e) If Q is orthogonal, then $\det(Q)$ is .
- (f) The linear system $A\mathbf{x} = \mathbf{b}$ is consistent if and only if \mathbf{b} is orthogonal to .
- (g) If A is a symmetric 2×2 matrix with 3-eigenvector $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\det(A) = 6$.
Then A has -eigenvector .
- (h) The projection matrix for orthogonally projecting onto $\text{col}(A)$ is . [A has independent columns.]
If A is orthogonal, this simplifies to .
- (i) If W is the subspace of \mathbb{R}^6 of all solutions to $x_1 + 3x_4 - 2x_6 = 0$, then $\dim W =$, $\dim W^\perp =$.

(extra scratch paper)