Homework Set 8 (Lecture 27)

Problem 8

Example 10. Determine the 2×2 matrix Q for rotation by 49 degrees.

Solution. Recall that the rotation matrix for angle θ is $Q = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ (see Example 142 in Lecture 27).

Hence, $Q = \begin{bmatrix} \cos(49^\circ) & -\sin(49^\circ) \\ \sin(49^\circ) & \cos(49^\circ) \end{bmatrix} = \begin{bmatrix} \cos\left(49 \cdot \frac{2\pi}{360}\right) & -\sin\left(49 \cdot \frac{2\pi}{360}\right) \\ \sin\left(49 \cdot \frac{2\pi}{360}\right) & \cos\left(49 \cdot \frac{2\pi}{360}\right) \end{bmatrix} \approx \begin{bmatrix} 0.6561 & -0.7547 \\ 0.7547 & 0.6561 \end{bmatrix}$.

Problem 9

Example 11. Determine the square of the norm of the vector $\begin{bmatrix} 5-5i \\ 1+3i \end{bmatrix}$.

Solution. $\left\| \begin{bmatrix} 5-5i \\ 1+3i \end{bmatrix} \right\|^2 = |5-5i|^2 + |1+3i|^2 = (5^2+(-5)^2) + (1^2+3^2) = 50+10 = 60$

Problem 10

Example 12. Given $A = \begin{bmatrix} 2-i & -1+i \\ 3-3i & 2+3i \end{bmatrix}$, determine A^*.

Solution. $A^* = (\bar{A})^T = \begin{bmatrix} 2+i & -1-i \\ 3+3i & 2-3i \end{bmatrix}^T = \begin{bmatrix} 2+i & 3+3i \\ -1-i & 2-3i \end{bmatrix}$

Problem 11

Example 13. Invert the complex number $5-4i$.

Solution. $\frac{1}{5-4i} = \frac{5+4i}{(5-4i)(5+4i)} = \frac{5+4i}{5^2-(4i)^2} = \frac{5+4i}{25+16} = \frac{5}{41} + \frac{4}{41}i$

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