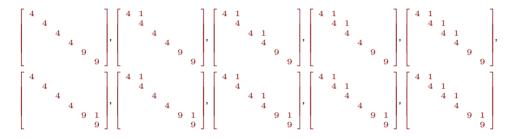### Problem 5

Example 7. How many different Jordan normal forms are there for a 6×6 matrix with eigenvalues $4, 4, 4, 4, 9, 9$?

Solution. One eigenvalue has multiplicity 4, the other multiplicity 2.

Therefore, there are $5 \cdot 2 = 10$ possible Jordan normal forms.

[See Example 132 in Lecture 25 for where the 5 and 2 come from. Alternatively, you can see this from the listing that follows below.]

Listing all of them.

$$
\begin{bmatrix} 4 & & & & & \\ & 4 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 9 & \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 9 & \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & & & & \\ & & 4 & 1 & & \\ & & & 4 & & \\ & & & & 9 & \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & 1 & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 9 & \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & 1 & & & \\ & & 4 & 1 & & \\ & & & 4 & & \\ & & & & 9 & \\ & & & & & 9 \end{bmatrix},
$$

$$
\begin{bmatrix} 4 & & & & & \\ & 4 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 9 & 1 \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 9 & 1 \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & & & & \\ & & 4 & 1 & & \\ & & & 4 & & \\ & & & & 9 & 1 \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & 1 & & & \\ & & 4 & & & \\ & & & 4 & & \\ & & & & 9 & 1 \\ & & & & & 9 \end{bmatrix},
\begin{bmatrix} 4 & 1 & & & & \\ & 4 & 1 & & & \\ & & 4 & 1 & & \\ & & & 4 & & \\ & & & & 9 & 1 \\ & & & & & 9 \end{bmatrix},
$$

Problem 6

Example 8. How many different Jordan normal forms are there for a 14×14 matrix with eigenvalues $3, 4, 4, 4, 4, 5, 5, 5, 5, 7, 7, 7, 7, 8$?

Solution. The multiplicities of the eigenvalues are $1, 4, 4, 4, 1$.

Hence, there are $1 \cdot 5 \cdot 5 \cdot 5 \cdot 1 = 125$ possible Jordan normal forms.