Homework Set 8 (Lecture 24)

Problem 4

Example 6. Write the (third-order) differential equation $y''' = 5y'' + 6y' + 2y$ as a system of (first-order) differential equations.

Solution. Write $\boldsymbol{y} = \begin{bmatrix} y \\ y' \\ y'' \end{bmatrix}$. Then $\boldsymbol{y}' = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 6 & 5 \end{bmatrix} \boldsymbol{y}$.

[Note how the third row of the matrix encodes $y''' = 2y + 6y' + 5y''$, while the first and second row encode the (trivial) equations $y' = y'$ and $y'' = y''$.].

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