Homework Set 7 (Lecture 22)

Problem 5

You find the answer to this problem at the very beginning of Lecture 22.

Problem 6

Example 1. Let A be the matrix for reflecting through the plane spanned by $\begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix}$. Diagonalize A as $A = PDP^{-1}$.

Solution. The eigenvalues of A are $1, 1, -1$. The 1-eigenspace of A is W, and the -1-eigenspace is $W^{\perp}$.

If $W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} \right\}$, then $W^{\perp} = \operatorname{null}\left(\begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & -3 \end{bmatrix} \right)$.

It follows from $\begin{bmatrix} 1 & 4 & -2 \\ 0 & 2 & -3 \end{bmatrix} \overset{\frac{1}{2}R_2 \Rightarrow R_2}{\rightsquigarrow} \begin{bmatrix} 1 & 4 & -2 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix} \overset{R_1 - 4R_2 \Rightarrow R_1}{\rightsquigarrow} \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & -\frac{3}{2} \end{bmatrix}$ that $W^{\perp} = \operatorname{span}\left\{ \begin{bmatrix} -4 \\ 3/2 \\ 1 \end{bmatrix} \right\}$.

We can therefore choose $D = \begin{bmatrix} 1 & & \\ & 1 & \\ & & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 0 & -4 \\ 4 & 2 & 3/2 \\ -2 & -3 & 1 \end{bmatrix}$.

Comment. From here, we can produce a diagonalization of the form $A = PDP^T$. How?

Problem 7

Example 2. Solve the initial value problem $y' = 3y$ with $y(0) = 8$.

Solution. The general solution to $y' = 3y$ is $y(t) = Ce^{3t}$.
Since $y(0) = Ce^0 = C = 8$, we conclude that the unique solution to the IVP is $y(t) = 8e^{3t}$.

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