Assessment Quiz #1

MATH 316 — Linear Algebra II
Wednesday, Feb 6

Please print your name:

Problem 1. We want to find values for the parameters a, b, c such that $z = a + bx + c\ln(y)$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Solution. The equations $a + bx_i + b\ln(y_i) = z_i$ translate into the system:

$$\underbrace{\begin{bmatrix} 1 & x_1 & \ln(y_1) \\ 1 & x_2 & \ln(y_2) \\ 1 & x_3 & \ln(y_3) \\ \vdots & \vdots & \vdots \end{bmatrix}}_{A} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \underbrace{\begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ \vdots \end{bmatrix}}_{\boldsymbol{z}}$$

Of course, this is usually inconsistent. To find the best possible a, b, c we compute a least squares solution. □

Problem 2. Write down a precise definition of what it means for vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ to be linearly independent.

Solution. Vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ are linearly independent if and only if the only solution to

$$x_1\boldsymbol{v}_1 + x_2\boldsymbol{v}_2 + \ldots + x_m\boldsymbol{v}_m = \boldsymbol{0}$$

is the trivial one ($x_1 = x_2 = \ldots = x_m = 0$). □

Problem 3. Fill in the blanks.

(a) Let $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$. A basis for null$(A)$ is ______.

(b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} =$ ______

(c) $\hat{\boldsymbol{x}}$ is a least squares solution of $A\boldsymbol{x} = \boldsymbol{b}$ if and only if ______.

(d) col(A) is the orthogonal complement of ______. null(A) is the orthogonal complement of ______.

(e) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to ______.

(f) The projection matrix for orthogonally projecting onto col(A) is $P =$ ______..

(g) If W is the space of all solutions to $x_1 + 2x_2 + x_3 - x_4 = 0$, then $\dim W =$ ______ and $\dim W^{\perp} =$ ______.

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Solution.

(a) $A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \end{bmatrix}$. A basis for null$(A)$ is $\begin{bmatrix} -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ -4 \\ 1 \end{bmatrix}$.

(b) $\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

(c) $\hat{\boldsymbol{x}}$ is a least squares solution of $A\boldsymbol{x} = \boldsymbol{b}$ if and only if $A^T A\boldsymbol{x} = A^T\boldsymbol{b}$.

(d) col(A) is the orthogonal complement of null(A^T). null(A) is the orthogonal complement of row(A).

(e) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to null(A^T).

(f) The projection matrix for orthogonally projecting onto col(A) is $P = A(A^T A)^{-1} A^T$.

(g) If W is the subspace of $\mathbb{R}^4$ of all solutions to $x_1 + 2x_2 + x_3 - x_4 = 0$, then $\dim W = 3$ and $\dim W^\perp = 1$. □

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