Midterm #1

MATH 316 — Linear Algebra II
Friday, Feb 15

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (6 points)

(a) Find the least squares solution to $\begin{bmatrix} 1 & 2 \\ 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} -1 \\ 0 \\ 2 \\ 5 \end{bmatrix}$.

(b) Determine the least squares line for the data points $(2,-1), (1,0), (1,2), (-1,5)$.

Problem 2. (9 points)

(a) Using Gram–Schmidt, obtain an orthonormal basis for $W = \operatorname{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix} \right\}$.

(b) Determine the orthogonal projection of $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$ onto W.

(c) Determine the QR decomposition of the matrix $\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 0 & -1 \end{bmatrix}$.

(d) Determine a basis for the orthogonal complement $W^{\perp}$.

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Problem 3. (3 points) We want to find values for the parameters a, b, c such that $z = ax + bx^2 + c\ln(y)$ best fits some given points $(x_1, y_1, z_1), (x_2, y_2, z_2), \ldots$ Set up a linear system such that $[a, b, c]^T$ is a least squares solution.

Problem 4. (2 points) Write down a precise definition of what it means for vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ to be linearly independent.

Vectors $\boldsymbol{v}_1, \boldsymbol{v}_2, \ldots, \boldsymbol{v}_m \in \mathbb{R}^n$ are linearly independent if and only if ...

Problem 5. (10 points) Fill in the blanks.

(a) Let $A = \begin{bmatrix} 1 & 5 & -2 & 0 & -4 \\ 0 & 0 & 0 & 1 & 3 \end{bmatrix}$. A basis for null($A$) is ______.

(b) null(A) is the orthogonal complement of ______. col(A) is the orthogonal complement of ______.

(c) If A is a 5×7 matrix with rank 4, then $\dim \operatorname{col}(A) =$ ______ and $\dim \operatorname{null}(A) =$ ______.

(d) By definition, a matrix Q is orthogonal if and only if ______.

(e) If Q is orthogonal, then $\det(Q)$ is ______.

(f) The linear system $A\boldsymbol{x} = \boldsymbol{b}$ is consistent if and only if $\boldsymbol{b}$ is orthogonal to ______.

(g) If A is a symmetric 2×2 matrix with 3-eigenvector $\begin{bmatrix} 1 \\ 4 \end{bmatrix}$ and $\det(A) = 6$.

Then A has ______-eigenvector ______.

(h) The projection matrix for orthogonally projecting onto col(A) is $P =$ ______.

If A is orthogonal, this simplifies to ______.

(i) Let W be the subspace of $\mathbb{R}^5$ of all solutions to $x_1 - x_3 + 2x_5 = 0$. $\dim W =$ ______ and $\dim W^{\perp} =$ ______.

(extra scratch paper)