

Preparing for the Final

Please print your name:

Bonus challenge. Let me know about any typos you spot in the posted solutions (or lecture sketches). Any typo, that is not yet fixed by the time you send it to me, is worth a bonus point.

Problem 1. The final exam will be comprehensive, that is, it will cover the material of the whole semester.

- (a) Redo the practice problems for both midterms.
- (b) Retake both midterms. (For your convenience, these are posted with and without solutions.)
- (c) Redo the online homeworks #7 and #8.
- (d) Do the problems below. (Solutions will be posted soon.)

Problem 2. Solve the initial value problem

$$\mathbf{y}' = \begin{bmatrix} 0 & -2 \\ -4 & 2 \end{bmatrix} \mathbf{y}, \quad \mathbf{y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Problem 3.

- (a) Convert the third-order differential equation

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order differential equations.

- (b) Solve the original differential equation by solving the system.

Problem 4.

- (a) What are the possible Jordan normal forms of a 6×6 matrix with eigenvalues 7, 7, 3, 3, 3, 3?
- (b) How many different Jordan normal forms are there for a 10×10 matrix with eigenvalues 8, 6, 6, 2, 2, 2, 1, 1, 1, 1?

Problem 5. Find the best approximation of $f(x) = x$ on the interval $[0, 4]$ using a function of the form $y = a + b\sqrt{x}$.

Problem 6. Give a basis for the space of all polynomials $p(x)$ of degree 4 or less such that $p(0) = p(1)$ and $p'(-1) = 0$.