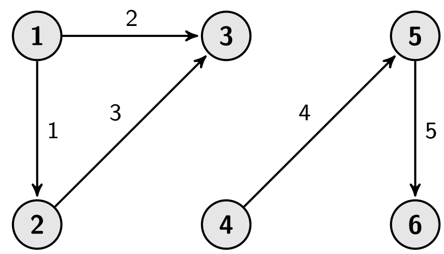
Quiz #4

MATH 316 — Linear Algebra II
Friday, Apr 14

Please print your name:

Problem 1. Let M be the edge-node incidence matrix of the graph to the right. For this problem, do not write down M.



(a) Give a basis for $\text{null}(M)$.

(b) Give a basis for $\text{null}(M^T)$.

Solution.

(a) A basis for $\text{null}(M)$ is: $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ (one vector for each connected component)

(b) A basis for $\text{null}(M^T)$ is: $\begin{bmatrix} 1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$ (one vector for the loop $1 \to 2 \to 3$) $\square$

Problem 2. Suppose the internet consists of only the three webpages A, B, C which link to each other as indicated in the diagram. Rank these webpages by computing their PageRank vector.

Solution. Recall that we model a random surfer, who randomly clicks on links. Let a_t be the probability that such a surfer will be on page A at time t. Likewise, b_t, c_t are the probabilities that the surfer will be on page B or C.

The transition probabilities are as follows.

$$\begin{bmatrix} a_{t+1} \\ b_{t+1} \\ c_{t+1} \end{bmatrix} = \begin{bmatrix} 0 \cdot a_t + 1 \cdot b_t + 1 \cdot c_t \\ \frac{1}{2} \cdot a_t + 0 \cdot b_t + 0 \cdot c_t \\ \frac{1}{2} \cdot a_t + 0 \cdot b_t + 0 \cdot c_t \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 & 1 \\ \frac{1}{2} & 0 & 0 \\ \frac{1}{2} & 0 & 0 \end{bmatrix}}_{=T} \begin{bmatrix} a_t \\ b_t \\ c_t \end{bmatrix}$$

To find the equilibrium state, we determine an appropriate 1-eigenvector of the transition matrix T.

The 1-eigenspace is $\text{null}(T - 1 \cdot I) = \text{null}\left(\begin{bmatrix} -1 & 1 & 1 \\ \frac{1}{2} & -1 & 0 \\ \frac{1}{2} & 0 & -1 \end{bmatrix} \right)$, which has basis $\begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$.

The corresponding equilibrium state is $\frac{1}{4} \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.5 \\ 0.25 \\ 0.25 \end{bmatrix}$. This is the PageRank vector.

We therefore rank A highest, and B, C lower (tied). $\square$

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