Midterm #2

MATH 316 — Linear Algebra II
Wednesday, Mar 29

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (8 points) Solve the initial value problem $\boldsymbol{y}' = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \boldsymbol{y}, \quad \boldsymbol{y}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}.$

Armin Straub
straub@southalabama.edu

Problem 2. (8 points) Consider $A = \begin{bmatrix} 2 & 2 \\ -1 & -1 \end{bmatrix}$.

(a) Determine the SVD of A.

(b) Determine the pseudoinverse of A.

Problem 3. (2 points) Carefully state what the spectral theorem says about symmetric (real) matrices.

Armin Straub
straub@southalabama.edu

Problem 4. (2 points) Convert the third-order differential equation

$$y''' = 3y'' - y' + 2y, \quad y(0) = 1, \quad y'(0) = 3, \quad y''(0) = 0$$

to a system of first-order differential equations. Do not solve the equation.

Problem 5. (2 points) What are the possible Jordan normal forms of a 4×4 matrix with eigenvalues $2, 2, 3, 3$?

Problem 6. (8 points) Fill in the blanks.

(a) Let A be the 3×3 matrix for an orthogonal projection onto a 2-dimensional subspace.

Then $\det(A) =$ ______, and the eigenvalues of A are ______.

(b) If $A = \begin{bmatrix} i & 1+2i \\ 3 & 4 \end{bmatrix}$, then its conjugate transpose is $A^* =$ ______.

(c) The norm of the vector $\boldsymbol{v} = \begin{bmatrix} 1+i \\ 1 \end{bmatrix}$ is $\|\boldsymbol{v}\| =$ ______.

(d) By Euler's identity, $e^{i\theta} =$ ______.

(e) If A is $n \times n$, then the product of its singular values equals ______.

(f) The pseudoinverse of $A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \end{bmatrix}$ is $A^+ =$ ______.

(g) If A is invertible, then $A^+ =$ ______.

(h) Suppose the linear system $A\boldsymbol{x} = \boldsymbol{b}$ has infinitely many solutions $\boldsymbol{x}$.

Which of these solutions is produced by $A^+\boldsymbol{b}$? ______

Armin Straub
straub@southalabama.edu

(extra scratch paper)