Preparing for Midterm #2

MATH 316 — Linear Algebra II
Wednesday, Mar 29

Please print your name:

Problem 1.

(a) Do the practice problems that were compiled from the examples from our lectures.

In particular, fill in all the conceptual empty boxes.

To save time, you don't need to work through all details. However, make sure that you know how to do each problem.

(b) Retake Quiz 3.

(c) Do the problems below. (Solutions will be posted soon.)

Bonus challenge. Let me know about any typos you spot in our lecture sketches or the posted solutions (surely, there should be some). Any typo that is not yet fixed on our course website by the time you send it to me, is worth a small bonus.

Problem 2. Consider $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix}$.

(a) Determine the SVD of A.

(b) Determine the pseudoinverse of A.

(c) Find the smallest solution to $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$.

(Then, as a mild check, compare its norm to the obvious solution $\boldsymbol{x} = [\ 1\ \ 1\ \ 0\ ]^T$.)

(d) What is the best approximation to A using a 2×3 matrix with rank 1?

Problem 3. Solve the initial value problem

$$\boldsymbol{y}' = \begin{bmatrix} 0 & -2 \\ -4 & 2 \end{bmatrix} \boldsymbol{y}, \qquad \boldsymbol{y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}.$$

Problem 4.

(a) Convert the third-order differential equation

$$y''' = 6y'' - 3y' - 10y, \quad y(0) = 1, \quad y'(0) = 2, \quad y''(0) = 3$$

to a system of first-order differential equations.

(b) Solve the original differential equation by solving the system.

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Problem 5. True or false? (As usual, "true" means that the statement is always true.) Explain!

(a) The product of two orthogonal matrices is orthogonal.

(b) $A^T A$ is symmetric for any matrix A.

(c) $A A^T$ is symmetric for any matrix A.

(d) A real $n \times n$ matrix A has real eigenvalues.

(e) The determinant of A is equal to the product of the singular values of A.

(f) The determinant of A is equal to the product of the eigenvalues of A.

(g) If the matrix A is symmetric, then $\langle A\boldsymbol{v}, \boldsymbol{w} \rangle = \langle \boldsymbol{v}, A\boldsymbol{w} \rangle$ for all vectors $\boldsymbol{v}, \boldsymbol{w}$.

(h) If the matrix A is orthogonal, then $\langle A\boldsymbol{v}, A\boldsymbol{w} \rangle = \langle \boldsymbol{v}, \boldsymbol{w} \rangle$ for all vectors $\boldsymbol{v}, \boldsymbol{w}$.

(i) If $\boldsymbol{v}$ and $\boldsymbol{w}$ are eigenvectors of A with different eigenvalues, then $\langle \boldsymbol{v}, \boldsymbol{w} \rangle = \mathbf{0}$.

(j) A is invertible if and only if the only solution to $A\boldsymbol{x} = \mathbf{0}$ is $\boldsymbol{x} = \mathbf{0}$.

(k) An $n \times n$ matrix A has eigenvalue 0 if and only if it has singular value 0.

(l) An $n \times n$ matrix A has eigenvalue 1 if and only if it has singular value 1.

(m) An $n \times n$ matrix A is singular if and only if 0 is an eigenvalue of A.

(n) An $n \times n$ matrix A is singular if and only if 0 is a singular value of A.

(o) Every symmetric real $n \times n$ matrix A is diagonalizable.

(p) Every symmetric real $n \times n$ matrix A is invertible.

Problem 6.

(a) If A has λ-eigenvalue $\boldsymbol{v}$, then A^3 has $\boxed{}$.

(b) A is singular if and only if $\boxed{\dim \operatorname{null}(A) }$.

(c) The eigenvalues of a 5×5 matrix for orthogonally projecting onto a 3-dimensional subspace are $\boxed{}$.

(d) Suppose A is the 3×3 matrix of a reflection through a plane (containing the origin).

Then $\det(A) = \boxed{}$, and the eigenvalues of A are $\boxed{}$.

(e) What exactly does it mean for a matrix A to have full column rank?

(f) Precisely state the spectral theorem.

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