Any serious linear algebra problems are done by a machine. Let us see how to use the open-source computer algebra system **Sage** to do basic computations for us.

Sage is freely available at sagemath.org. Instead of installing it locally (it's huge!) we can conveniently use it in the cloud at cloud.sagemath.com from any browser.

Sage is built as a **Python** library, so any Python code is valid. Here, we will just use it as a fancy calculator.

Let us see how to define and work with vectors:

```
Sage] v = vector([1,2,3])
```

```
Sage] w = vector([1,1,1])
```

```
Sage] 7*v - 3*w
```

$$(4, 11, 18)$$

```
Sage] v.norm()
```

$$\sqrt{14}$$

```
Sage] v.dot_product(w)
```

$$6$$

In a very similar way, we can work with matrices:

```
Sage] A = matrix([[4,0,2],[2,2,2],[1,0,3]])
```

```
Sage] A
```

$$\begin{bmatrix} 4 & 0 & 2 \\ 2 & 2 & 2 \\ 1 & 0 & 3 \end{bmatrix}$$

```
Sage] A.transpose()
```

$$\begin{bmatrix} 4 & 2 & 1 \\ 0 & 2 & 0 \\ 2 & 2 & 3 \end{bmatrix}$$

```
Sage] A.rank()
```

3

```
Sage] A.determinant()
```

$$20$$

```
Sage] A.inverse()
```

$$\begin{bmatrix} \frac{3}{10} & 0 & -\frac{1}{5} \\ -\frac{1}{5} & \frac{1}{2} & -\frac{1}{5} \\ -\frac{1}{10} & 0 & \frac{2}{5} \end{bmatrix}$$

```
Sage] A.rref()
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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Recall that we diagonalized the matrix A in Example 12. Sage can, of course, do that for us:

```
Sage] A.eigenvalues()
```

$$[5, 2, 2]$$

```
Sage] A.eigenvectors_right()
```

$$\left[\left(5, \left[\left(1, 1, \frac{1}{2}\right)\right], 1\right), (2, [(1, 0, -1), (0, 1, 0)], 2)\right]$$

```
Sage] A.eigenmatrix_right()
```

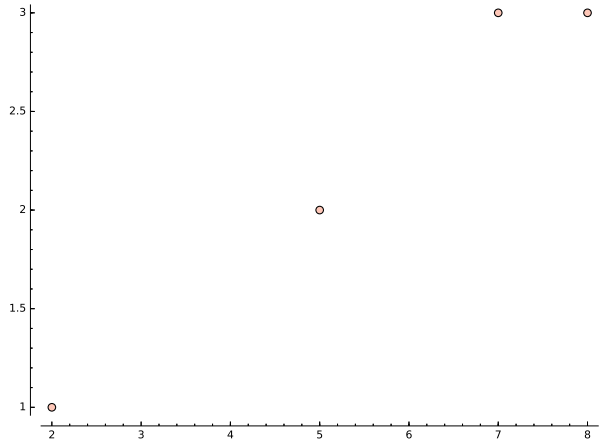
$$\left(\begin{bmatrix} 5 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ \frac{1}{2} & -1 & 0 \end{bmatrix}\right)$$

Which line best fits the data points $(2,1), (5,2), (7,3), (8,3)$?

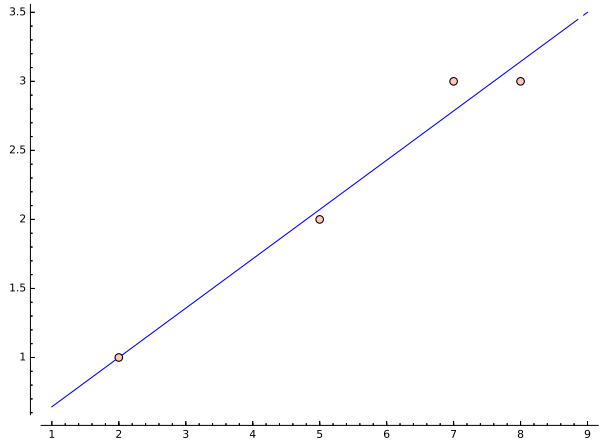
Questions like this can be answered using **linear regression**, which we will soon discuss.

We learn how to determine that the best fit is given by $y = \frac{2}{7} + \frac{5}{14}x$. Let's just look at the pictures for now.

```
Sage] points = [(2,1),(5,2),(7,3),(8,3)]
Sage] scatter_plot(points)
```




```
Sage] scatter_plot(points) + plot(2/7+5/14*x,1,9)
```



Example 33. (homework) Check out what is happening in the following computations. We have played with the matrix A in Example 30.

Note that **kernel** is another word for null space. $\mathrm{null}(A)$ is `A.right_kernel()` in Sage.

```
Sage] A = matrix([[1,2,1],[2,4,0],[3,6,0]])
```

```
Sage] A.right_kernel()
```

$$\mathrm{RowSpan}_{\mathbb{Z}}\begin{bmatrix} 2 & -1 & 0 \end{bmatrix}$$

```
Sage] A.left_kernel()
```

$$\mathrm{RowSpan}_{\mathbb{Z}}\begin{bmatrix} 0 & 3 & -2 \end{bmatrix}$$

```
Sage] A.column_space()
```

$$\mathrm{RowSpan}_{\mathbb{Z}}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix}$$

```
Sage] A.row_space()
```

$$\mathrm{RowSpan}_{\mathbb{Z}}\begin{bmatrix} 1 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We can also solve equations.

Note how we are getting only a particular solution. What is the general solution to $A\boldsymbol{x} = \boldsymbol{b}$?

```
Sage] b = vector([3,-2,-3])
```

```
Sage] A\b
```

$$(-1,\, 0,\, 4)$$

```
Sage] A.solve_right(b)
```

$$(-1,\, 0,\, 4)$$

Revisit Example 32 with the help of Sage.