Quiz #5

MATH 237 — Linear Algebra I
Tuesday, Nov 1

Please print your name:

Problem 1. Find the eigenvalues of $A = \begin{bmatrix} 4 & 2 & 1 \\ 0 & 3 & 0 \\ 1 & 2 & 4 \end{bmatrix}$ as well as bases for the eigenspaces.

Solution. By expanding by the second row, we find that the characteristic polynomial is

$$\begin{vmatrix} 4-\lambda & 2 & 1 \\ 0 & 3-\lambda & 0 \\ 1 & 2 & 4-\lambda \end{vmatrix} = (3-\lambda) \begin{vmatrix} 4-\lambda & 1 \\ 1 & 4-\lambda \end{vmatrix} = (3-\lambda)[(4-\lambda)^2 - 1] = (3-\lambda)(\lambda-3)(\lambda-5).$$

Hence, the eigenvalues are $\lambda = 3$ (with multiplicity 2) and $\lambda = 5$.

- For $\lambda = 5$, the eigenspace $\operatorname{null}\left(\begin{bmatrix} -1 & 2 & 1 \\ 0 & -2 & 0 \\ 1 & 2 & -1 \end{bmatrix} \right)$ has basis $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.
- For $\lambda = 3$, the eigenspace $\operatorname{null}\left(\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & 0 \\ 1 & 2 & 1 \end{bmatrix} \right)$ has basis $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$.

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