

Quiz #4

Please print your name:

Problem 1. Let $A = \begin{bmatrix} 1 & -2 & 0 & 2 & 3 \\ 1 & -2 & 1 & 0 & 0 \\ 2 & -4 & 0 & 4 & 6 \\ 1 & -2 & 1 & 0 & 0 \end{bmatrix}$. Find a basis for each of $\text{col}(A)$, $\text{row}(A)$ and $\text{null}(A)$.

Solution. We eliminate:

$$\begin{bmatrix} 1 & -2 & 0 & 2 & 3 \\ 1 & -2 & 1 & 0 & 0 \\ 2 & -4 & 0 & 4 & 6 \\ 1 & -2 & 1 & 0 & 0 \end{bmatrix} \begin{array}{l} R_2 - R_1 \Rightarrow R_2 \\ R_3 - 2R_1 \Rightarrow R_3 \\ R_4 - R_1 \Rightarrow R_4 \\ \rightsquigarrow \end{array} \begin{bmatrix} 1 & -2 & 0 & 2 & 3 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & -3 \end{bmatrix} \begin{array}{l} R_4 - R_2 \Rightarrow R_4 \\ \rightsquigarrow \end{array} \begin{bmatrix} 1 & -2 & 0 & 2 & 3 \\ 0 & 0 & 1 & -2 & -3 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The general solution to $A\mathbf{x} = \mathbf{0}$ is $\mathbf{x} = \begin{bmatrix} 2s_1 - 2s_2 - 3s_3 \\ s_1 \\ 2s_2 + 3s_3 \\ s_2 \\ s_3 \end{bmatrix}$.

• basis for $\text{col}(A)$: $\begin{bmatrix} 1 \\ 1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}$

• basis for $\text{row}(A)$: $\begin{bmatrix} 1 \\ -2 \\ 0 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ -2 \\ -3 \end{bmatrix}$

• basis for $\text{null}(A)$: $\begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -3 \\ 0 \\ 3 \\ 0 \\ 1 \end{bmatrix}$

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