Quiz #9

MATH 237 — Linear Algebra I
Thursday, Nov 5

Please print your name:

Problem 1.

(a) Is $W = \left\{ \begin{bmatrix} 3a-b \\ -a \\ a+b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ a vector space? If yes, find a basis.

(b) Is $W = \left\{ \begin{bmatrix} 3a-b \\ -1 \\ a+b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ a vector space? If yes, find a basis.

(c) Is $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a - 2b = 3 \right\}$ a vector space? If yes, find a basis.

(d) Is $W = \left\{ \begin{bmatrix} a \\ b \\ c \end{bmatrix} : a - 2b = 3c \right\}$ a vector space? If yes, find a basis.

Solution.

(a) Since $\begin{bmatrix} 3a-b \\ -a \\ a+b \end{bmatrix} = a \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix} + b \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$, we see that $W = \operatorname{span}\left\{ \begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

In particular, W is a vector space with basis $\begin{bmatrix} 3 \\ -1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$ (because these two vectors are independent).

(b) W is not a vector space because $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \notin W$.

(c) W is not a vector space because $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \notin W$.

(d) Writing $a - 2b = 3c$ as $a - 2b - 3c = 0$, we see that $W = \operatorname{null}(\begin{bmatrix} 1 & -2 & -3 \end{bmatrix})$.

In particular, W is a vector space. Since $A = \begin{bmatrix} 1 & -2 & -3 \end{bmatrix}$ is already in RREF, we can read off the general solution $\boldsymbol{x} = \begin{bmatrix} 2s_1 + 3s_2 \\ s_1 \\ s_2 \end{bmatrix}$ to $A\boldsymbol{x} = \mathbf{0}$. Hence, a basis for W is $\begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$. □

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