# Quiz #5

**MATH 237 — Linear Algebra I**
**Thursday, Sept 24**

***Please print your name:***

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**Problem 1.** Decide whether the following vectors are linearly independent. No computations necessary!

(a) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ ☐ dependent ☐ independent

(b) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}$ ☐ dependent ☐ independent

(c) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 0 \end{bmatrix}$ ☐ dependent ☐ independent

**Solution.**

(a) dependent (because of the zero vector)

(b) independent (because the two vectors are not multiples of each other)

(c) dependent (because these are four vectors in $\mathbb{R}^3$) ☐

**Problem 2.** Decide whether the vectors $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 4 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ are linearly independent.

**Solution.** We eliminate!

$$\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix} \underset{\rightsquigarrow}{R_3 - 2R_1 \Rightarrow R_3} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 0 & 2 & -4 \end{bmatrix} \underset{\rightsquigarrow}{R_3 \Leftrightarrow R_2} \begin{bmatrix} 1 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 0 & 1 \end{bmatrix}$$

Each column contains a pivot, so the system $\begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 1 \\ 2 & 4 & 0 \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ has no free variables and hence a unique solution (namely the trivial one, $\boldsymbol{x} = \mathbf{0}$). This means that the three vectors are linearly independent. ☐

Armin Straub
straub@southalabama.edu