

Quiz #2

Please print your name:

Problem 1. Convert the given system to an augmented matrix and then find all solutions by transforming the system to reduced echelon form and back substituting.

$$\begin{aligned} 2x_1 + x_2 &= 2 \\ -x_1 - x_2 - x_3 &= 1 \end{aligned}$$

Solution. We eliminate!

$$\begin{aligned} & \left[\begin{array}{ccc|c} 2 & 1 & 0 & 2 \\ -1 & -1 & -1 & 1 \end{array} \right] \xrightarrow{2R_2+R_1 \Rightarrow R_2} \left[\begin{array}{ccc|c} 2 & 1 & 0 & 2 \\ 0 & -1 & -2 & 4 \end{array} \right] \xrightarrow{R_1+R_2 \Rightarrow R_1} \left[\begin{array}{ccc|c} 2 & 0 & -2 & 6 \\ 0 & -1 & -2 & 4 \end{array} \right] \\ & \xrightarrow{R_1+R_2 \Rightarrow R_1} \left[\begin{array}{ccc|c} 2 & 0 & -2 & 6 \\ 0 & -1 & -2 & 4 \end{array} \right] \xrightarrow{\begin{array}{l} \frac{1}{2}R_1 \Rightarrow R_1 \\ -R_2 \Rightarrow R_2 \end{array}} \left[\begin{array}{ccc|c} 1 & 0 & -1 & 3 \\ 0 & 1 & 2 & -4 \end{array} \right] \end{aligned}$$

x_3 is a free variable, and we set $x_3 = s$, where s can be any number. The general solution is:

$$\begin{aligned} x_1 &= 3 + s \\ x_2 &= -4 - 2s \\ x_3 &= s \end{aligned}$$

Since we have plenty of time left, we verify our answer by plugging into the original system:

$$\begin{aligned} 2(3 + s) + (-4 - 2s) &\stackrel{\checkmark}{=} 2 \\ -(3 + s) - (-4 - 2s) - s &\stackrel{\checkmark}{=} 1 \end{aligned}$$

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