Homework #1

MATH 237 — Linear Algebra I
due Thursday, Aug 27, in class

Please print your name:

Problem 1. Consider the following system of linear equations:

$$\begin{array}{rcrcrcl} x_1 & + & 2x_2 & - & x_3 & = & 2 \\ 3x_1 & - & 2x_2 & + & x_3 & = & 2 \\ -x_1 & + & 2x_2 & + & 3x_3 & = & 4 \end{array}$$

(a) Determine its augmented matrix.

(b) Apply a sequence of elementary row operations to obtain an equivalent matrix in echelon form ("triangular" shape). Record all your row operations.

(c) Apply back-substitution to solve the corresponding linear system. Verify that your solution also solves the original linear system.

(d) Is this linear system consistent?

Solution.

(a) $\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & -2 & 1 & 2 \\ -1 & 2 & 3 & 4 \end{array}\right]$

(b)

$$\left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 3 & -2 & 1 & 2 \\ -1 & 2 & 3 & 4 \end{array}\right] \underset{R_3+R_1 \Rightarrow R_3}{\overset{R_2-3R_1 \Rightarrow R_2}{\rightsquigarrow}} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -8 & 4 & -4 \\ 0 & 4 & 2 & 6 \end{array}\right] \overset{R_3+\frac{1}{2}R_2 \Rightarrow R_3}{\rightsquigarrow} \left[\begin{array}{ccc|c} 1 & 2 & -1 & 2 \\ 0 & -8 & 4 & -4 \\ 0 & 0 & 4 & 4 \end{array}\right]$$

(c) We find $x_3 = 1$ (because the third equation is $4x_3 = 4$), then $x_2 = 1$ (because $-8x_2 + 4 \cdot 1 = -4$) and, finally, $x_1 = 1$ (because $x_1 + 2 \cdot 1 - 1 \cdot 1 = 2$).

To verify that your solution also solves the original linear system, we just plug-in the values $x_1 = 1$, $x_2 = 1$ and $x_3 = 1$:

$$\begin{array}{rcrcrcl} 1 & + & 2 \cdot 1 & - & 1 & \overset{\checkmark}{=} & 2 \\ 3 \cdot 1 & - & 2 \cdot 1 & + & 1 & \overset{\checkmark}{=} & 2 \\ -1 & + & 2 \cdot 1 & + & 3 \cdot 1 & \overset{\checkmark}{=} & 4 \end{array}$$

(d) Yes, the linear system is consistent (because it has at least one solution; we actually know that it has exactly one solution). □

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