Preparation problems for the discussion sections on October 21st and 23rd

1. *Let*

$$A = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & -1 \\ 1 & 0 & -1 \end{bmatrix}.$$

(1) *Draw a directed graph with numbered edges and nodes, whose edge-node incidence matrix is A.*

(2) *Find a basis for the solutions to $A\boldsymbol{x} = 0$ in two ways: by using the matrix A, and then by using a property of the graph.*

(3) *Find a basis for the solutions to $A^T\boldsymbol{y} = 0$ in two ways: by using the matrix A, and then by using a property of the graph.*

(4) *Conclude from the fundamental theorem that a vector $\boldsymbol{b}$ is in the column space of A if and only if it satisfies $b_1 + b_2 - b_3 = 0$. What does this condition mean when the b's are potential differences?*

(5) *Conclude from the fundamental theorem that a vector $\boldsymbol{f}$ is in the row space of A if and only if satisfies $f_1 + f_2 + f_3 = 0$. What does that mean when the f's are net currents into the nodes?*

2. *Consider the matrix*

$$A = \begin{bmatrix} -1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix}.$$

(1) *Draw a directed graph with numbered edges and nodes, whose edge-node incidence matrix is A.*

(2) *Use a property of the graph to find a basis for* $\mathrm{Nul}(A)$.

(3) *Use a property of the graph to find a basis for* $\mathrm{Nul}(A^T)$.

(4) *Find a basis for* $\mathrm{Col}(A^T)$ *by choosing a spanning tree of this graph.*