Preparation problems for the discussion sections on September 30th and October 2nd

1. *Find an explicit description of Nul A, where*

$$A = \begin{bmatrix} 1 & 3 & 5 & 0 \\ 0 & 1 & 4 & -2 \end{bmatrix}.$$

2. *Let* $A = \begin{bmatrix} 1 & 3 & 1 & 2 \\ 2 & 6 & 4 & 8 \\ 0 & 0 & 2 & 4 \end{bmatrix}$ *and* $\boldsymbol{b} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$.
Find the set of all solutions to the equation $A\boldsymbol{x} = \boldsymbol{b}$, *and express it as the sum of a particular solution and solutions in the null space of A.*

3. *Consider the subspace*

$$W := \{ \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix} \ : \ a = 2b + c, \ 2a = c - 3d \}.$$

Find a matrix A and a matrix B such that $W = Col(A)$ *and* $W = Nul(B)$.

4. *a) For which values of h is* $\boldsymbol{v}_3$ *in the span of* $\boldsymbol{v}_1$ *and* $\boldsymbol{v}_2$*? b) For which values of h is* $\{\boldsymbol{v}_1, \boldsymbol{v}_2, \boldsymbol{v}_3\}$ *linearly dependent?*

(i) $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ -5 \\ -3 \end{bmatrix}, \boldsymbol{v}_2 = \begin{bmatrix} -2 \\ 10 \\ 6 \end{bmatrix}, \boldsymbol{v}_3 = \begin{bmatrix} 2 \\ -9 \\ h \end{bmatrix},$

(ii) $\boldsymbol{v}_1 = \begin{bmatrix} -7 \\ 3 \\ -6 \end{bmatrix}, \boldsymbol{v}_2 = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}, \boldsymbol{v}_3 = \begin{bmatrix} 1 \\ 2 \\ h \end{bmatrix},$

(iii) $\boldsymbol{v}_1 = \begin{bmatrix} 6 \\ -4 \\ 3 \end{bmatrix}, \boldsymbol{v}_2 = \begin{bmatrix} 6 \\ -12 \\ 2 \end{bmatrix}, \boldsymbol{v}_3 = \begin{bmatrix} 9 \\ h \\ 3 \end{bmatrix}.$

5. *Check whether the following sets of vectors are linearly independent. Justify your answer!*

a) $\{ \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ -3 \\ 9 \end{bmatrix} \}$ *b)* $\{ \begin{bmatrix} 4 \\ -2 \end{bmatrix}, \begin{bmatrix} 6 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -2 \\ 1 \end{bmatrix} \}$

c) $\{ \begin{bmatrix} 4 \\ -2 \\ 6 \end{bmatrix}, \begin{bmatrix} 6 \\ 2 \\ 9 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \}$ *d)* $\{ \begin{bmatrix} 4 \\ 0 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ -5 \\ 5 \\ 10 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ -1 \\ -11 \end{bmatrix} \}$

6. *True or false? Justify your answers!*

(a) *If three vectors in* $\mathbb{R}^3$ *span a plane, then they are linearly dependent.*
(b) *If a set contains fewer vectors than there are entries in the vectors, then the set is linearly independent.*
(c) *If* $\boldsymbol{x}$ *and* $\boldsymbol{y}$ *are linearly independent, and if* $\boldsymbol{z}$ *is in the span of* $\boldsymbol{x}$ *and* $\boldsymbol{y}$, *then* $\boldsymbol{x}$, $\boldsymbol{y}$ *and* $\boldsymbol{z}$ *are linearly dependent.*
(d) *If a set in* $\mathbb{R}^n$ *is linearly independent, then it contains n vectors.*