Preparation problems for the discussion sections on September 9th and 11th

1. *Determine if the vector* $\begin{bmatrix} -5 \\ 11 \\ -7 \end{bmatrix}$ *is a linear combination of* $\begin{bmatrix} 1 \\ -2 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 8 \end{bmatrix}$.

2. *Give a geometric description of* $\textbf{\textit{Span}}\{\begin{bmatrix} 3 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} -2 \\ 0 \\ 3 \end{bmatrix}\}$.

3. *True or false? Justify your answers!*

(a) *Let A be an $m \times n$-matrix and B be an $m \times l$-matrix. Then the product AB is defined.*

(b) *The weights $c_1, ..., c_p$ in a linear combination $c_1\boldsymbol{v}_1 + ... + c_p\boldsymbol{v}_p$ cannot all be zero.*

(c) $\textbf{\textit{Span}}\{\boldsymbol{u}, \boldsymbol{v}\}$ *contains the line through $\boldsymbol{u}$ and the origin.*

(d) *Asking whether the linear system corresponding to $\begin{bmatrix} \boldsymbol{a}_1 & \boldsymbol{a}_2 & \boldsymbol{a}_3 & \boldsymbol{b} \end{bmatrix}$ is consistent, is the same as asking whether $\boldsymbol{b}$ is a linear combination of $\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3$.*

4. *Determine whether* $\begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ *is a linear combination of the columns of* $\begin{bmatrix} 1 & 0 & 5 \\ -2 & 1 & -6 \\ 0 & 2 & 8 \end{bmatrix}$.

5. *Compute AB in two ways: a) by the definition, where $A\boldsymbol{b}_1$ and $A\boldsymbol{b}_2$ are calculated separately, and b) by the row-column rule for computing AB.*

i) $A = \begin{bmatrix} 4 & -2 \\ -3 & 0 \\ 3 & 5 \end{bmatrix}, B = \begin{bmatrix} 1 & 3 \\ 2 & -1 \end{bmatrix}$ *ii)* $A = \begin{bmatrix} 5 & 1 & 0 \\ 6 & 0 & 1 \end{bmatrix}, B = \begin{bmatrix} 6 & 1 \\ 0 & -1 \\ 0 & 1 \end{bmatrix}$

6. *Let* $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$.

(1) *If* $\boldsymbol{x} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$, *what is $A\boldsymbol{x}$?*

(2) *If* $\boldsymbol{x} = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, *what is $A\boldsymbol{x}$?*

(3) *Is $A\boldsymbol{x} = \boldsymbol{b}$ uniquely solvable: is there for a given $\boldsymbol{b}$ always exactly one $\boldsymbol{x}$?*

7. *(Some interesting matrices) Find a matrix A (what size!) such that*

- $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

- $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} x \\ y+3x \end{bmatrix}.$

- $A\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} y \\ x \end{bmatrix}$