Midterm #2

Please print your name:

No notes, fancy calculators or tools of any kind are permitted. There are 31 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (3 points) Let $y(x)$ be the unique solution to the IVP $y'' = (x+1)y^2 + 1$, $y(0) = 2$, $y'(0) = 3$.

Determine the first several terms (up to x^3) in the power series of $y(x)$.

Problem 2. (3 points) A mass-spring system is described by the DE $my'' + 7y = F(t)$ where $F(t)$ is an external force with period 3. For which values of m can resonance occur?

Problem 3. (3 points) Find a minimum value for the radius of convergence of a power series solution to

$$(x-3)y'' = \frac{2y+1}{x^2+1} \quad \text{at } x = 1.$$

Problem 4. (6 points) Derive a recursive description of a power series solution $y(x)$ (around $x=0$) to the differential equation $y''=3y-x^2y'$.

Problem 5. (4 points) Consider the function $f(t)=2t-1$, defined for $t\in[0,1]$. Sketch the following for $t\in[-3,3]$.

(a) Fourier series of $f(t)$ (b) Fourier cosine series of $f(t)$ (c) Fourier sine series of $f(t)$

In each sketch, carefully mark the values of the Fourier series at discontinuities.
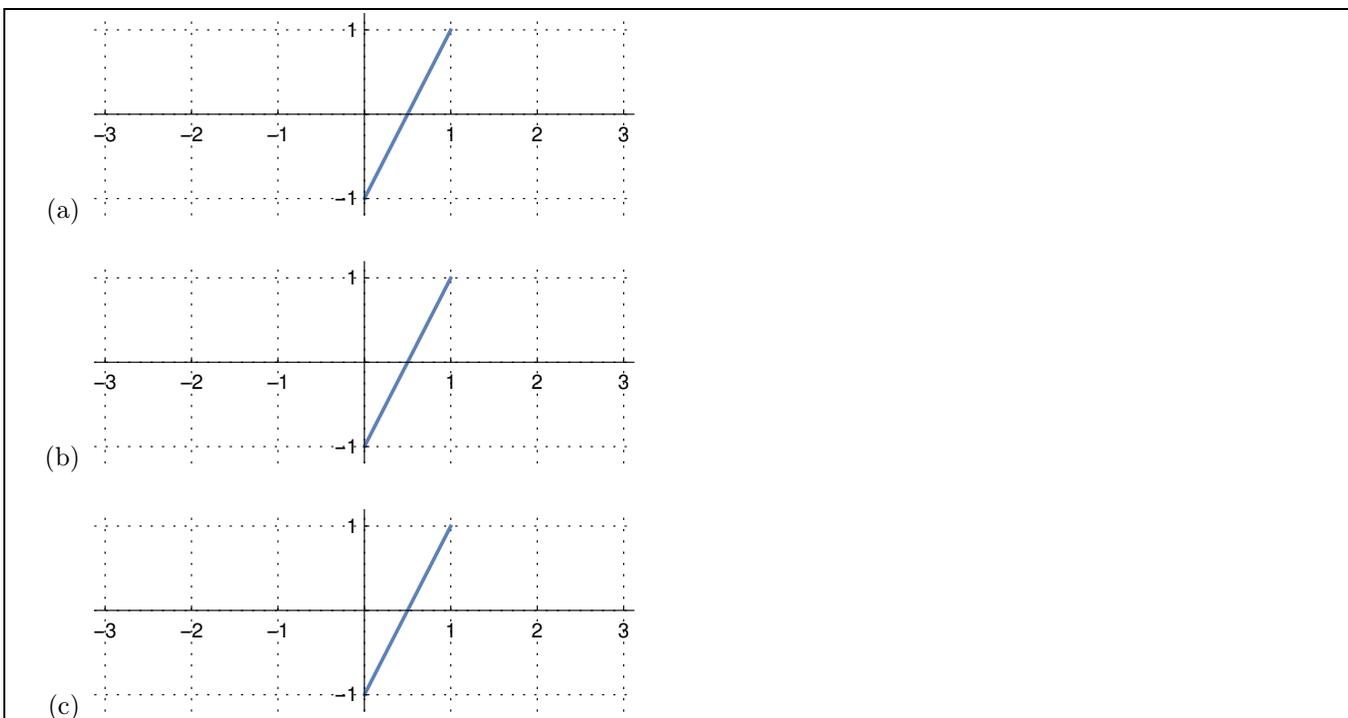
Problem 6. (7 points) Determine the equilibrium points of the system $\frac{dx}{dt} = xy - 3$, $\frac{dy}{dt} = x(y+1)$ and classify their stability.

Problem 7. (5 points)

(a) By Euler's identity, we have $e^{ix} =$

(b) Determine the power series around $x = 0$: $\quad \dfrac{5}{1+2x} =$

(c) Determine the power series around $x = 0$: $\quad 3e^{-x} =$

(d) Suppose $y(x) = \displaystyle\sum_{n=0}^{\infty} a_n (x+1)^n$. How can we compute the a_n from $y(x)$? $\quad a_n =$

(e) If $f(t) = \displaystyle\sum_{n=1}^{\infty} b_n \sin\left(\frac{n\pi t}{5}\right)$, then we can compute the b_n as $b_n =$

(extra scratch paper)

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