Midterm #1

MATH 332 — Differential Equations II
Friday, Feb 21, 2025

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (10 points) Let $M = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$.

(a) Compute e^{Mt}.

(b) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 0 \\ 6 \end{bmatrix}$.

(c) Determine all equilibrium points of $\begin{bmatrix} x \\ y \end{bmatrix}' = M \begin{bmatrix} x \\ y \end{bmatrix}$ and their stability.

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Problem 2. (8 points) Fill in the blanks. None of the problems should require any computation!

(a) Determine a (homogeneous linear) recurrence equation satisfied by $a_n = (3n+2)4^n + 7$.

You can use the operator N to write the recurrence. No need to simplify, any form is acceptable.

(b) Let y_p be any solution to the inhomogeneous linear differential equation $y'' - 9y = 4xe^x - 5e^{2x}$. Find a homogeneous linear differential equation which y_p solves.

You can use the operator D to write the DE. No need to simplify, any form is acceptable.

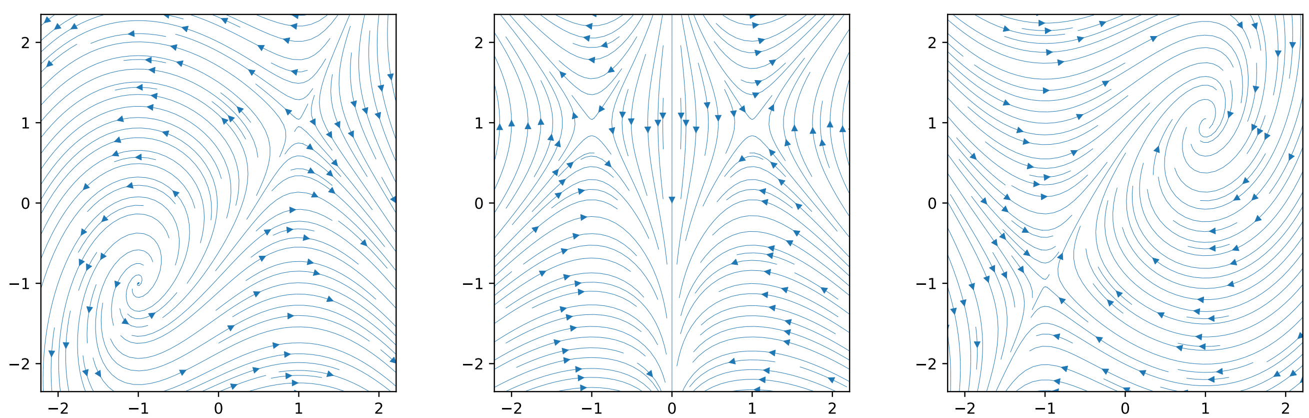
(c) Consider a homogeneous linear differential equation with constant real coefficients which has order 4. Suppose $y(x) = 3x - 5e^{2x}\cos(x)$ is a solution. Write down the general solution.

(d) If $M^n = \begin{bmatrix} 2-3^n & -2+2\cdot 3^n \\ 1-3^n & -1+2\cdot 3^n \end{bmatrix}$, then $e^{Mx} =$ ______.

Problem 3. (3 points)

(a) Circle the phase portrait below which belongs to $\frac{\mathrm{d}x}{\mathrm{d}t} = x - y$, $\frac{\mathrm{d}y}{\mathrm{d}t} = 1 - x^2$.

(b) Determine all equilibrium points and classify the stability of each.



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Problem 4. (3 points) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' - 4y_1' &= 3y_2 \\ y_2'' + 2y_2 &= 5y_1' \end{aligned} \qquad y_1(0) = 7,\ y_1'(0) = 1,\ y_2(0) = 2,\ y_2'(0) = 0$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y},\ \boldsymbol{y}(0) = \boldsymbol{c}$.

Problem 5. (1+4+1 points) Consider the sequence a_n defined by $a_{n+2} = a_{n+1} + 2a_n$ and $a_0 = 1$, $a_1 = 8$.

(a) The next two terms are $a_2 =$ ☐ and $a_3 =$ ☐.

(b) A Binet-like formula for a_n is $a_n =$ ☐, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$ ☐.

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(extra scratch paper)