

Rational power series

EG $y(x) = \frac{1}{1-x-x^2}$

Write $y(x)$ as a power series and determine the coefficients.

$$y(x) = \sum_{n=0}^{\infty} a_n x^n$$

$$\begin{aligned} 1 &= (1-x-x^2) \sum_{n=0}^{\infty} a_n x^n \\ &= \sum_{n=0}^{\infty} a_n x^n - \sum_{n=0}^{\infty} a_n x^{n+1} - \sum_{n=0}^{\infty} a_n x^{n+2} \\ &= \sum_{n=0}^{\infty} a_n x^n - \sum_{n=1}^{\infty} a_{n-1} x^n - \sum_{n=2}^{\infty} a_{n-2} x^n \end{aligned}$$

compare coefficients of x^n :

$$n=0 : 1 = a_0$$

$$n=1 : 0 = a_1 - a_0$$

$$n \geq 2 : 0 = a_n - a_{n-1} - a_{n-2}$$

$$a_0 = 1$$

$$a_1 = 1$$

$$a_n = a_{n-1} + a_{n-2}$$

Fibonacci numbers!

$$\frac{1}{1-x-x^2} = 1 + x + 2x^2 + 3x^3 + 5x^4 + 8x^5 + \dots$$

note

$$\begin{aligned} 0 &= a_{n+2} - a_{n+1} - a_n \\ &= (N^2 - N - 1) a_n \end{aligned}$$

fun

$$\frac{1000000}{998999} = 1.001002003005008013021 \dots$$

$$\frac{1}{1-x-x^2} \text{ with } x = \frac{1}{1000}$$