Systems of recurrences

EG $\quad a_{n+2} = a_{n+1} + 2a_n \qquad a_0 = 1,\ a_1 = 8$

Write as system of (order 1) recurrences.

write: $\quad b_n = a_{n+1}$

translate to $\quad \begin{cases} a_{n+1} = b_n \\ b_{n+1} = 2a_n + b_n \end{cases}$

matrix form: $\quad \begin{bmatrix} a_{n+1} \\ b_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} a_n \\ b_n \end{bmatrix} \qquad \begin{bmatrix} a_0 \\ b_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 8 \end{bmatrix}$

$$\vec{a}_{n+1} = M \vec{a}_n \qquad \vec{a}_0 = \vec{c}$$

has unique solution $\vec{a}_n = M^n \vec{c}$.

matrix powers

EG $\quad a_{n+3} - 4a_{n+2} + a_{n+1} + 6a_n = 0$

Write as a system.

write $b_n = a_{n+1}$, $\quad c_n = a_{n+2}$

translates to $\quad \begin{cases} a_{n+1} = b_n \\ b_{n+1} = c_n \\ c_{n+1} = -6a_n - b_n + 4c_n \end{cases}$

matrix form:

$$\begin{bmatrix} a_{n+1} \\ b_{n+1} \\ c_{n+1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -1 & 4 \end{bmatrix} \begin{bmatrix} a_n \\ b_n \\ c_n \end{bmatrix}$$

$$\vec{a}_{n+1} = M \vec{a}_n$$