Linear DEs

DE $$y^{(n)} + p_{n-1}(x)\, y^{(n-1)} + \dots + p_1(x)\, y' + p_0(x)\, y = f(x)$$

order n

differential operators $\quad D := \frac{d}{dx}$

$$L := D^n + p_{n-1}(x) D^{n-1} + \dots + p_1(x) D + p_0(x)$$

this is a linear operator: $\quad L(y_1 + y_2) = Ly_1 + Ly_2$

$$L(\underbrace{c}_{\text{scalar}}\, y_1) = c\, L y_1$$

$$Ly = f(x)$$ inhomogeneous linear DE

linear algebra: $Ax = b$

corresponding homogeneous DE:

HDE $$Ly = 0$$ $\qquad Ax = \vec{0}$

- If y_1 and y_2 solve HDE then so does $c_1 y_1 + c_2 y_2$.
  $Ly_1 = 0 \quad Ly_2 = 0$
  $$L(c_1 y_1 + c_2 y_2) = c_1 \underbrace{L(y_1)}_{0} + c_2 \underbrace{L(y_2)}_{0} = 0$$
- There are n solutions $y_1, y_2, \dots, y_n$ of HDE so that the general solution of HDE is
  $$c_1 y_1 + c_2 y_2 + \dots + c_n y_n.$$

DE $$Ly = f(x)$$

- If y_p solves DE and y_h is the general solution of HDE, then the general solution of DE is
  $$y_p + y_h$$
  particular solution $\quad Ly_p = f \quad Ly_h = 0$
  $$L(y_p + y_h) \stackrel{!}{=} f$$
  $$= \underbrace{L(y_p)}_{f} + \underbrace{L(y_h)}_{0}$$