

Midterm #2

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 32 points in total. You need to show work to receive full credit.

Good luck!

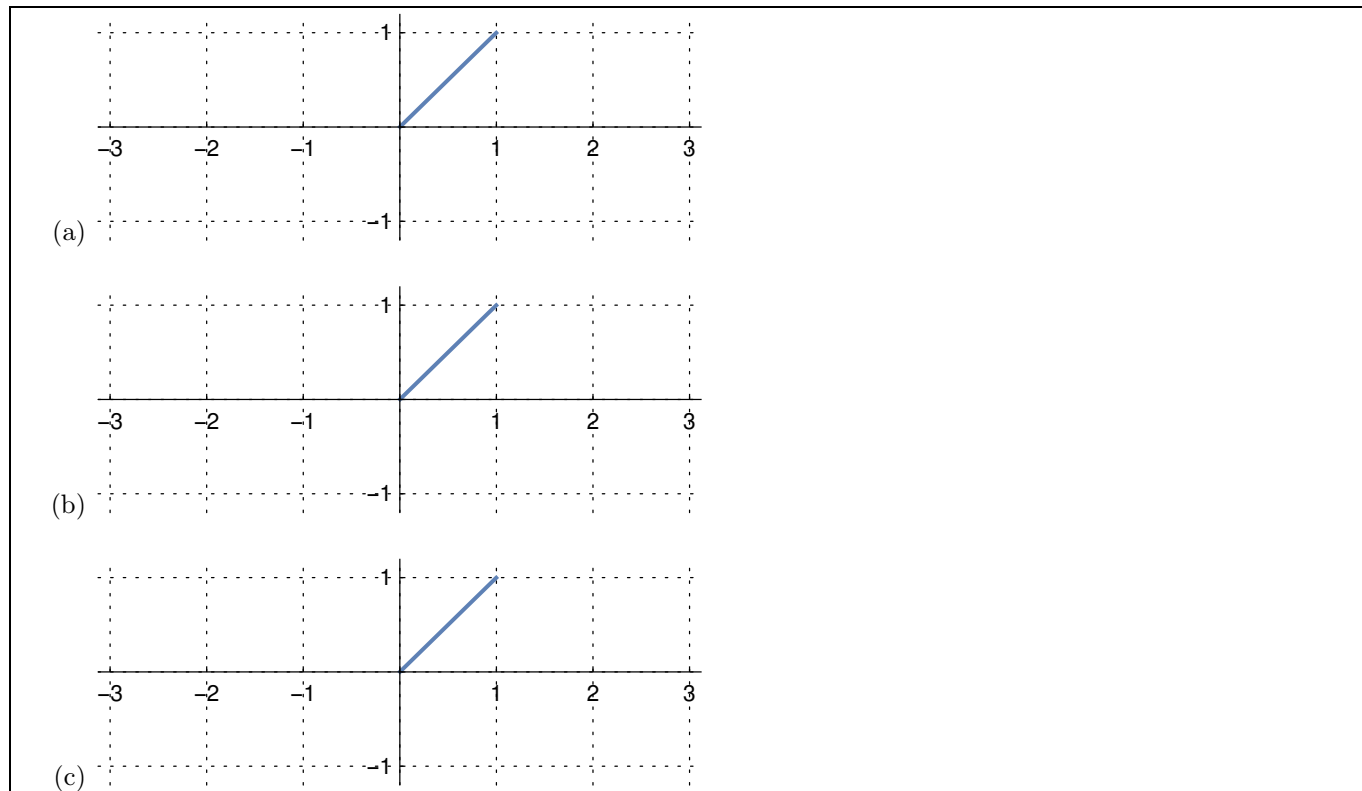
Problem 1. (5 points) Derive a recursive description of a power series solution $y(x)$ of the DE $y'' = (3x^2 - 2)y$.

Problem 2. (3 points) Derive a recursive description of the power series for $y(x) = \frac{1}{1 - 3x + 2x^2}$.

Problem 3. (4 points) Consider the function $f(t) = t$, defined for $t \in [0, 1]$.

- (a) Sketch the Fourier series of $f(t)$ for $t \in [-3, 3]$.
- (b) Sketch the Fourier cosine series of $f(t)$ for $t \in [-3, 3]$.
- (c) Sketch the Fourier sine series of $f(t)$ for $t \in [-3, 3]$.

In each sketch, carefully mark the values of the Fourier series at discontinuities.



Problem 4. (5 points)

(a) Suppose $y(x) = \sum_{n=0}^{\infty} a_n(x - x_0)^n$. How can we compute the a_n from $y(x)$? $a_n =$

(b) Suppose $f(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left(a_n \cos\left(\frac{n\pi t}{3}\right) + b_n \sin\left(\frac{n\pi t}{3}\right) \right)$. How can we compute the a_n and b_n from $f(t)$?

$a_n =$ and $b_n =$

(c) Determine the power series around $x = 0$: $e^{7x} =$

(d) Determine the power series around $x = 0$: $\frac{1}{1+x^2} =$

Problem 5. (3 points) A mass-spring system is described by the equation $y'' + ky = \sum_{n=1}^{\infty} \frac{1}{n^2 + 7} \cos\left(\frac{nt}{4}\right)$.

For which values of k does resonance occur?

Problem 6. (3 points) Find a minimum value for the radius of convergence of a power series solution to

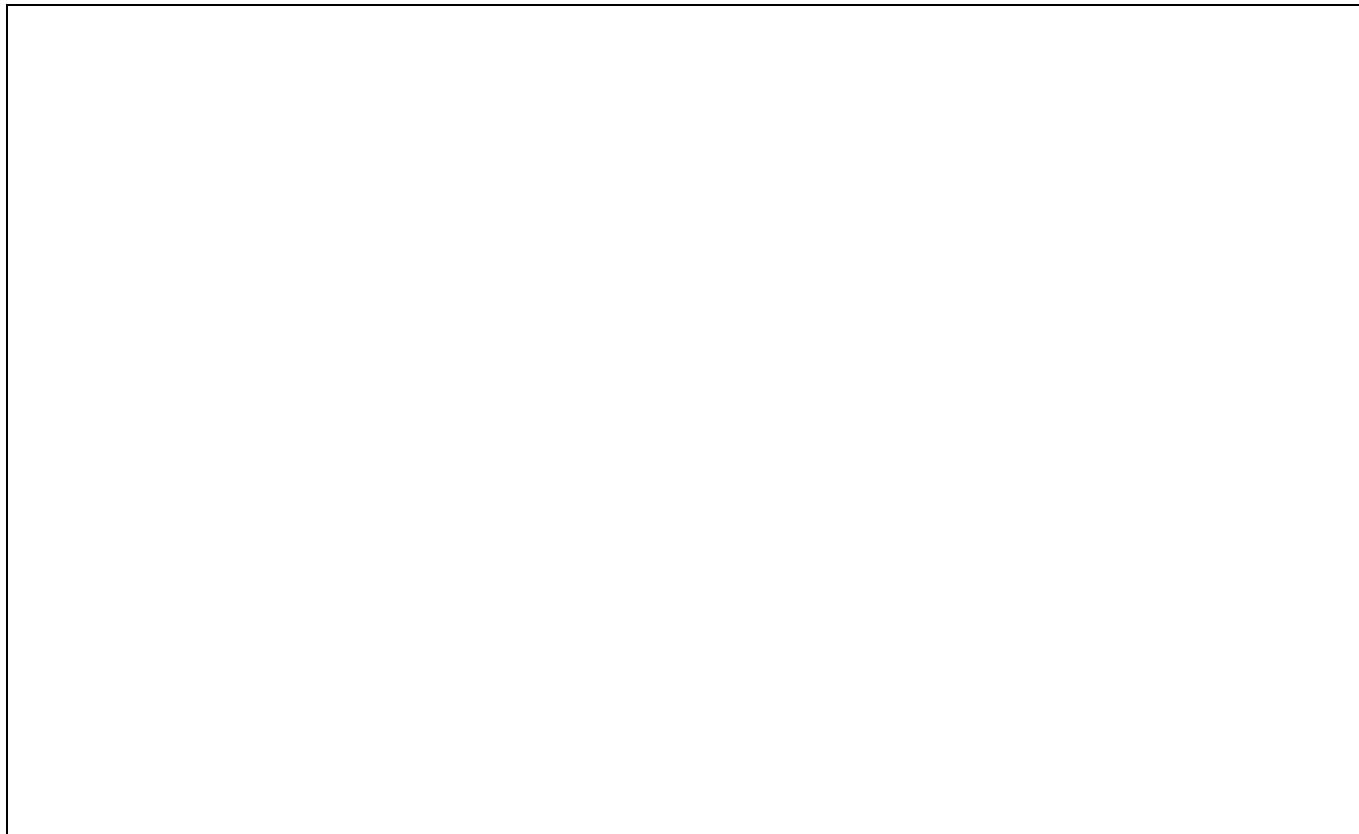
$$(x^2 + 1)y'' = \frac{y}{x + 1} \quad \text{at } x = 2.$$

Problem 7. (2 points) Suppose that the matrix A satisfies $e^{Ax} = \begin{bmatrix} 3e^x - 2e^{2x} & -6e^x + 6e^{2x} \\ e^x - e^{2x} & -2e^x + 3e^{2x} \end{bmatrix}$.

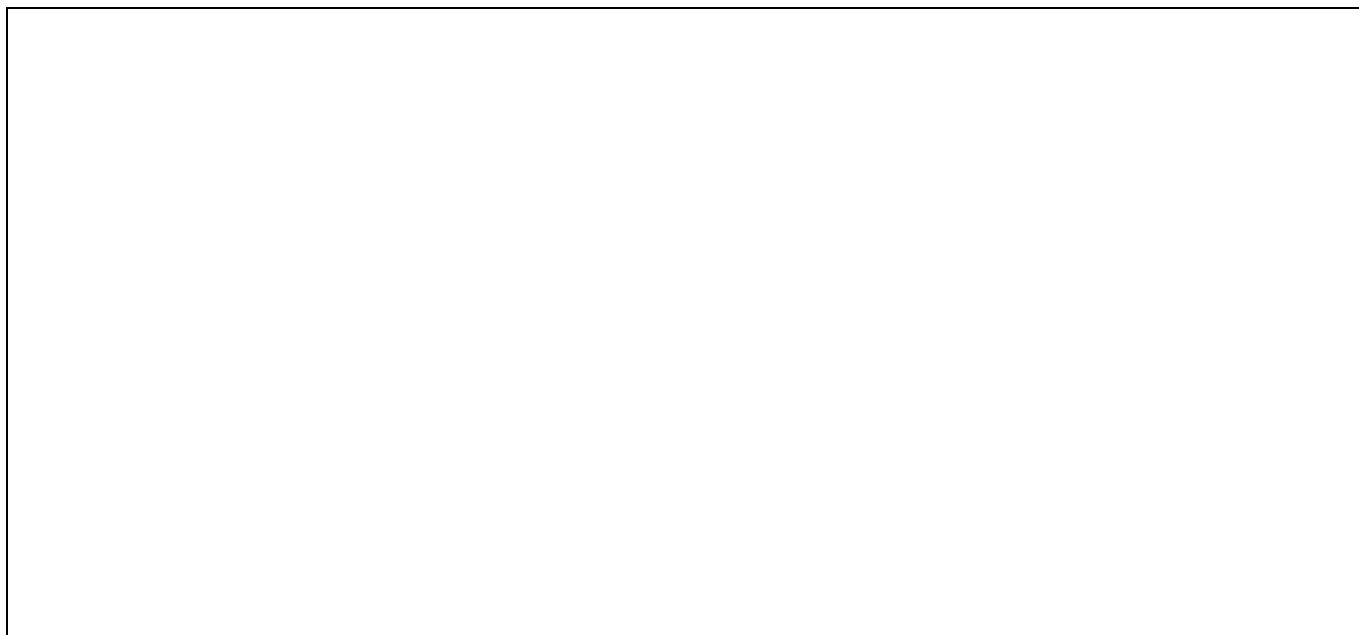
(a) $\mathbf{y}' = A\mathbf{y}$, $\mathbf{y}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is solved by $\mathbf{y}(x) =$.

(b) $A =$

Problem 8. (4 points) Find all eigenfunctions and eigenvalues of $y'' + \lambda y = 0$, $y(0) = 0$, $y(2) = 0$ in the case $\lambda > 0$.



Problem 9. (3 points) Let $y(x)$ be the unique solution to the IVP $y'' = x + 3y^2$, $y(0) = 2$, $y'(0) = 1$. Determine the first several terms (up to x^3) in the power series of $y(x)$.



(extra scratch paper)