Midterm #1

MATH 332 — Differential Equations II
Tuesday, Oct 1

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 30 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (3 points) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' &= 3y_1' - 5y_2 \\ y_2'' &= y_1' - y_2' + 3y_1 \end{aligned} \qquad y_1(0) = -2,\ y_1'(0) = 1,\ y_2(0) = 0,\ y_2'(0) = 3$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y}$, $\boldsymbol{y}(0) = \boldsymbol{c}$.

$M =$	$\boldsymbol{c} =$

Problem 2. (3 points) Determine a (homogeneous linear) recurrence equation satisfied by $a_n = (n+2)3^n - 7$.
Write the recurrence in explicit form (for instance, $a_{n+2} = a_{n+1} + a_n$ for the Fibonacci numbers).

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Problem 3. (9 points) Let $M = \begin{bmatrix} 5 & 4 \\ 8 & 1 \end{bmatrix}$.

(a) Compute e^{Mx}.

(b) Solve the initial value problem $\boldsymbol{y}' = M\boldsymbol{y}$ with $\boldsymbol{y}(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$.

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Problem 4. (1+4+1 points) Consider the sequence a_n defined by $a_{n+2} = 2a_{n+1} + 3a_n$ and $a_0 = -2$, $a_1 = 6$.

(a) The next two terms are $a_2 =$ $\boxed{}$ and $a_3 =$ $\boxed{}$.

(b) A Binet-like formula for a_n is $a_n =$ $\boxed{}$, and $\lim_{n\to\infty} \frac{a_{n+1}}{a_n} =$ $\boxed{}$.

Problem 5. (2 points) Consider a homogeneous linear differential equation with constant real coefficients which has order 5. Suppose $y(x) = 4x^2e^{-x} + e^{3x}\sin(2x)$ is a solution. Write down the general solution.

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Problem 6. (4 points) Find the general solution to $y'' - 4y = 3e^{2x} + 5$.

Problem 7. (3 points) Let y_p be any solution to the inhomogeneous linear differential equation $x^2y'' - y = e^{2x}$. Find a homogeneous linear differential equation which y_p solves. *Hint:* Do not attempt to solve the DE.

(extra scratch paper)