Midterm #2

Please print your name:

No notes, calculators or tools of any kind are permitted.   There are 36 points in total.   You need to show work to receive full credit.

Good luck!

Problem 1. (6 points) The mixtures in two tanks T_1, T_2 are kept uniform by stirring. Brine containing 3 lb of salt per gallon enters T_1 at a rate of 3 gal/min, while brine containing 2 lb of salt per gallon enters T_2 at a rate of 4gal/min. Mixed solution from T_1 is pumped into T_2 at a rate of 1 gal/min, and also from T_2 into T_1 at a rate of 2 gal/min. Initially, tank T_1 is filled with 10 gal water containing 5 lb salt, and tank T_2 with 20 gal pure water.

Denote by $y_i(t)$ the amount (in pounds) of salt in tank T_i at time t (in minutes). Derive a system of linear differential equations for the y_i, including initial conditions. (Do *not* attempt to solve the system.)

Problem 2. (4 points) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' + 3y_2 &= y_1' + 5 \\ y_2'' + 2y_1 &= 7y_2' \end{aligned} \qquad y_1(0) = 4, \ y_1'(0) = -1, \ y_2(0) = 0, \ y_2'(0) = 7$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y} + \boldsymbol{f}$, $\boldsymbol{y}(0) = \boldsymbol{c}$.

Problem 3. (3 points) The position $y(t)$ of a certain mass on a spring is described by $my'' + 5y = \cos(t) - 2\sin(3t)$. For which values of m, if any, does resonance occur?

Problem 4. (10 points) Determine the general solution of the following system: $\begin{array}{rcl} y_1' &=& y_1 + y_2 \\ y_2' &=& 3y_1 - y_2 + 9e^{-x} \end{array}$

Problem 5. (4 points) Assume that the angle $\theta(t)$ of a swinging pendulum is described by $\theta'' + 9\theta = 0$. Suppose $\theta(0) = 2$, $\theta'(0) = -6$. What are the period and the amplitude of the resulting oscillations?

Problem 6. (2 points) The motion of a certain mass on a spring is described by $y'' + dy' + 5y = 0$ where $d > 0$. For which values of d is the motion underdamped?

Problem 7. (7 points) Fill in the blanks. None of the problems should require any computation!

(a) Write down a homogeneous linear differential equation satisfied by $y(x) = 5 - 2x \sinh(4x) + (3x^2 - 1)e^x$.

Here, and in the next part, you can use the operator D to write the DE. No need to simplify, any form is acceptable.

(b) Let y_p be any solution to the inhomogeneous linear differential equation $y'' - 4y = x^2 - 5e^{2x}$. Find a homogeneous linear differential equation which y_p solves.

(c) Consider a homogeneous linear differential equation with constant real coefficients which has order 4. Suppose $y(x) = 2x - 5e^x \sin(3x)$ is a solution. Write down the general solution.

(d) Name the method which we can use to solve the differential equation $y'' - 4y = \frac{1}{x}$.

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(extra scratch paper)

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