Midterm #2

MATH 238 — Differential Equations I
Wednesday, Nov 12, 2025

Please print your name:

No notes, calculators or tools of any kind are permitted. There are 36 points in total. You need to show work to receive full credit.

Good luck!

Problem 1. (4 points) Assume that the angle $\theta(t)$ of a swinging pendulum is described by $\theta'' + 4\theta = 0$. Suppose $\theta(0) = 3$, $\theta'(0) = 8$. What are the period and the amplitude of the resulting oscillations?

Problem 2. (4 points) Consider the following system of initial value problems:

$$\begin{aligned} y_1'' - 3y_1 &= y_2 + 8 \\ y_2'' + 4y_2 &= 2y_1 - 7y_1' \end{aligned} \qquad y_1(0) = 2,\ y_1'(0) = 0,\ y_2(0) = 1,\ y_2'(0) = 6$$

Write it as a first-order initial value problem in the form $\boldsymbol{y}' = M\boldsymbol{y} + \boldsymbol{f}$, $\boldsymbol{y}(0) = \boldsymbol{c}$.

Problem 3. (3 points) The position $y(t)$ of a certain mass on a spring is described by $2y'' + ky = 5\cos(t) - \cos(3t)$. For which values of k, if any, does resonance occur?

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Problem 4. (10 points) Determine the general solution of the following system: $\begin{aligned} y_1' &= y_1 + 4y_2 - 6e^{3x} \\ y_2' &= y_1 - 2y_2 \end{aligned}$

Problem 5. (2 points) The motion of a certain mass on a spring is described by $my'' + y' + 3y = 0$ where $m > 0$. For which values of m is the motion overdamped?

Problem 6. (6 points) The mixtures in two tanks T_1, T_2 are kept uniform by stirring. Brine containing 2 lb of salt per gallon enters T_1 at 5 gal/min, and the solution is pumped at a rate of 4 gal/min into T_2. Finally, solution is leaving T_2 at 4 gal/min. Initially, T_1 and T_2 contain 40gal of pure water each.

Denote by $y_i(t)$ the amount (in pounds) of salt in tank T_i at time t (in minutes). Derive a system of linear differential equations for the y_i, including initial conditions. (Do *not* attempt to solve the system.)

Problem 7. (7 points) Fill in the blanks. None of the problems should require any computation!

(a) Let y_p be any solution to the inhomogeneous linear differential equation $y'' + 5y = 4 - 2x\,e^{3x}$. Find a homogeneous linear differential equation which y_p solves.

Here, and in the next part, you can use the operator D to write the DE. No need to simplify, any form is acceptable.

(b) Write down a homogeneous linear differential equation satisfied by $y(x) = (2+3x)\,e^{-x} - 3x^2$.

(c) Consider a homogeneous linear differential equation with constant real coefficients which has order 4. Suppose $y(x) = 5e^{2x}\cos(3x) + 7x\,e^{x}$ is a solution. Write down the general solution.

(d) Name the method which we can use to solve the differential equation $y'' + y' - 6y = 4\ln(x)$.

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(extra scratch paper)