

used in
MD5, SHA-1/2, ...

Merkle-Damgard:
compression function \rightarrow hash

$$\tilde{H}: \begin{matrix} b+c \\ \text{bits} \end{matrix} \rightarrow \begin{matrix} b \\ \text{bits} \end{matrix}$$

$$H: \begin{matrix} \text{any} \\ \text{bits} \end{matrix} \rightarrow \begin{matrix} b \\ \text{bits} \end{matrix}$$

THM: \tilde{H} collision-free \Rightarrow H collision-free

$$H(x) : x = \underbrace{x_1}_c \underbrace{x_2}_c \dots \underbrace{x_n}_c$$

$$h_1 = 0 \quad \text{or any other ...}$$

$$\underbrace{h_{i+1}}_b = \tilde{H}(\underbrace{h_i}_b, \underbrace{x_i}_c)$$

$$\Rightarrow \underline{h_{n+1}} =: H(x)$$

EG

x	000	001	010	011	100	101	110	111
$\tilde{H}(x)$	00	10	11	01	10	00	01	11

compressed 3 \rightarrow 2
 $\begin{matrix} 2+1 \\ b+c \end{matrix}$ b

$$H(1101) = 2 \quad \text{using MD with } h_1 = 0$$

$$x = \begin{matrix} x_1 & x_2 & x_3 & x_4 \\ 1 & 1 & 0 & 1 \end{matrix}$$

$$h_1 = 00$$

$$h_2 = \tilde{H}(h_1, x_1) = \tilde{H}(00, 1) = 10$$

$$h_3 = \tilde{H}(h_2, x_2) = \tilde{H}(10, 1) = 00$$

$$h_4 = \tilde{H}(h_3, x_3) = \tilde{H}(00, 0) = 00$$

$$h_5 = \tilde{H}(h_4, x_4) = \tilde{H}(00, 1) = 10 = H(1101)$$

example collision: $H(1) = H(1101)$