

# Linear congruential generators

LCG

fixed parameters  $a, b, m$   
from seed  $x_0$  generate

$$X_{n+1} \equiv aX_n + b \pmod{m}$$

$$\text{PRG}(x_0) = x_1 x_2 x_3 \dots$$

real world

glibc uses  $a = 1103515245$   $m = 2^{31}$   
 $b = 12345$

comments

- LCGs are easy to predict ( $\Rightarrow$  unsuitable for crypto)
- pros: easy to implement, decent statistical properties

EG

stream cipher using LCG  $X_{n+1} \equiv 5X_n + 3 \pmod{8}$

$$k = \underline{(100)}_2$$

$$m = (101\ 111\ 001)_2$$

$$x_0 = 4$$

$$x_1 = 5x_0 + 3 \equiv 7 \pmod{8}$$

$$x_2 = 5x_1 + 3 \equiv 6$$

$$x_3 = 5x_2 + 3 \equiv 1$$

$$\text{PRG}(x_0) = x_1 x_2 x_3 \dots$$

$$= 7, 6, 1, \dots$$

$$= 111, 110, 001, \dots$$

$$c = m \oplus \text{PRG}$$

$$\begin{array}{r} m \quad 101\ 111\ 001 \\ \oplus \text{PRG} \quad 111\ 110\ 001 \\ \hline = c \quad 010\ 001\ 000 \end{array}$$

EG

$$c = (111\ 111\ 111)_2$$

Eve knows: stream cipher with  $X_{n+1} \equiv 5X_n + 3 \pmod{8}$   
 $m = (110\ 1\dots)_2$

$$c = m \oplus \text{PRG}$$

$$\text{PRG} = c \oplus m$$

$$\begin{array}{r} 111\dots\ 1101\dots \\ = \underline{001000}\ 011 \\ x_1 = 1 \pmod{8} \end{array}$$

$$x_2 = 5x_1 + 3 \equiv 0 \pmod{8}$$

$$x_3 = 5x_2 + 3 \equiv 3$$

$$\begin{array}{r} c \quad 111\ 111\ 111 \\ \oplus \text{PRG} \quad 001\ 000\ 011 \\ \hline = m \quad 110\ 111\ 100 \end{array}$$