

Binary exponentiation

EG $3^{32} \pmod{101}$

$$3^2 = 9$$

$$3^4 = (3^2)^2 = 9^2 = 81 \equiv -20 \pmod{101}$$

$$3^8 = (3^4)^2 \equiv (-20)^2 = 400 \equiv -4$$

$$3^{16} = (3^8)^2 \equiv (-4)^2 = 16$$

$$3^{32} = (3^{16})^2 \equiv 16^2 = 256 \equiv 54 \pmod{101}$$

EG $3^{25} \pmod{101}$

$$25 = 16 + 8 + 1$$

$$3^{25} = 3^{16} \cdot 3^8 \cdot 3^1$$

$$\equiv 16 \cdot (-4) \cdot 3 = -192 \equiv 10 \pmod{101}$$

$$25 = 2^4 + 2^3 + 2^0$$

$$= 1 \cdot 2^4 + 1 \cdot 2^3 + 0 \cdot 2^2 + 0 \cdot 2^1 + 1 \cdot 2^0$$

$$= (11001)_2$$

base 2
= binary

EG $407^{37249} \pmod{101}$

$$\equiv 3^{37249}$$

$$\equiv 49 \pmod{100}$$

$$\equiv 3^{49} \equiv 3^{32} \cdot 3^{16} \cdot 3^1 \equiv 67 \pmod{101}$$

$$49 = 32 + 16 + 1$$