Euler's phi function

x invertible mod n $\iff \gcd(x,n)=1$

DEF $\phi(n)$ = # invertible residues mod n
= # of integers in $\{1, 2, \dots, n-1\}$ coprime to n

n

EG $\phi(p) = p-1$ "all residues mod p invertible except 0"

prime

EG $\phi(8) = 4 = 8 - \frac{8}{2}$ "all residues except multiples of 2"

invertible residues mod 8: 1, 3, 5, 7

EG $\phi(p^n) = p^n - \frac{p^n}{p} = p^n - p^{n-1}$ "all residues except multiples of p"

x invertible mod p^n
$\iff \gcd(x, p^n) = 1$
$\iff p \nmid x$ (p does not divide x)

THM ϕ is multiplicative:
$\phi(nm) = \phi(n)\phi(m)$ if $\gcd(n,m)=1$

CRT: $x \pmod{n \cdot m} \xleftrightarrow{\text{CRT}}$ $x \pmod n$, $x \pmod m$

EG $\phi(\underbrace{1000}_{2^3 \cdot 5^3}) = \phi(2^3) \cdot \phi(5^3) = (2^3 - 2^2)(5^3 - 5^2)$
$= (8-4)(125-25) = 400$

EG $\phi(\underbrace{980}_{2^2 \cdot 5 \cdot 7^2}) = \phi(2^2)\phi(5)\phi(7^2) = (2^2-2)(5-1)(7^2-7)$
$= 2 \cdot 4 \cdot 42 = 336$