

Credit card numbers: Luhn check

↖ Visa
4 2 6 6 8 3 4 2 8 4 1 2 9 2 7 0

8	12	16	8	16	2	18	14
8	2	3	6	7	3	8	2
7	4	2	2	9	2	5	0

x2
sum of digits

Luhn check sum of new digits $\equiv 0 \pmod{10}$

$$\underbrace{8+2+3+6}_9 + \underbrace{7+3+8+2}_9 + \underbrace{7+4+2+2}_4 + 9+2+5+0 \equiv 0 \pmod{10}$$

our credit card # passes Luhn check

purpose detect accidental errors

frequent errors:

- single digit incorrect
- two digits transposed

always detected

always detected, except 09 vs 90

EG

8 2 ...	vs	2 8 ...
7+2 = 9 (10)		4+8 = 2 (10)

Why? effect of doubling, sum-of-digits:

original	0	1	2	3	4	5	6	7	8	9
adjust	0	2	4	6	8	1	3	5	7	9
difference (mod 10)	0	1	2	3	4	6	7	8	9	0

$\equiv -4 \pmod{10}$