

## Homework Set 10

### Problem 1

**Example 25.** Consider the following compression function  $C(x)$  which takes three bits input and outputs two bits:

$x$	000	001	010	011	100	101	110	111
$C(x)$	10	00	11	01	01	10	00	11

Let  $H(x)$  be the hash function obtained from  $C(x)$  using the Merkle–Damgård construction (using initial value  $h_1 = 0$ ). Compute  $H(11000)$ .

**Solution.** Here,  $b = 2$  and  $c = 1$ , so that each  $x_i$  is 1 bit:  $x_1x_2x_3x_4x_5 = 11000$ .

$$h_1 = 00$$

$$h_2 = C(h_1, x_1) = C(001) = 00$$

$$h_3 = C(h_2, x_2) = C(001) = 00$$

$$h_4 = C(h_3, x_3) = C(000) = 10$$

$$h_5 = C(h_4, x_4) = C(100) = 01$$

$$h_6 = C(h_5, x_5) = C(010) = 11$$

Hence,  $H(11000) = h_6 = 11$ .

### Problem 2

**Example 26.** Bob's public RSA key is  $(N, e) = (35, 19)$ . His private key is  $d = 19$ . For signing, Bob uses the (silly) hash function  $H(x) = x \pmod{22}$ . Determine Bob's signature  $s$  of the message  $m = 361$ .

**Solution.**  $H(m) = 361 \pmod{22} = 9$ . The signature therefore is  $s = H(m)^d \pmod{N} = 9^{19} \equiv 9 \pmod{35}$ .

### Problem 3

**Example 27.** Alice uses an RSA signature scheme and the (silly) hash function  $H(x) = x_1 + x_2$ , where  $x_1 = 3x \pmod{11}$  and  $x_2 = 2x \pmod{29}$ , to sign the message  $m = 1299$  with the signature  $s = 121$ . Forge a second signed message.

**Solution.** Since we have no other information, in order to forge a signed message, we need to find another message with the same hash value as  $m = 1299$ . From our experience with the Chinese remainder theorem, we realize that changing  $x$  by  $11 \cdot 29$  does not change  $H(x)$ . Since  $1299 + 11 \cdot 29 = 1618$ , a second signed message is  $(1618, 121)$ .