

- AES-192/256 is first (and only) public cipher allowed by NSA for top secret information
- Unlike DES, AES is not a Feistel network.
While for a Feistel network, each round only encrypts half of the bits, all bits are being encrypted during each round. That's one indication why AES requires less rounds than DES.

5.3 Internals of AES

Each round consists of 4 **layers**. Each layer takes 128 bits input and outputs 128 bits in a reversible way (so that we can decrypt as long as we know the key). The 128 bit state consists of 16 bytes. These 16 bytes $c_{0,0}, c_{1,0}, c_{2,0}, c_{3,0}, c_{0,1}, \dots, c_{3,3}$ are often arranged in a 4x4 matrix as

$$\begin{bmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,3} \\ c_{1,0} & c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,0} & c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,0} & c_{3,1} & c_{3,2} & c_{3,3} \end{bmatrix}.$$

Each byte is identified with an element of $GF(2^8)$.

Example 139. $(0000\ 0101)_2$ represents the element $x^2 + 1$ in $GF(2^8)$.

The 4 layers are:

- **ByteSub**
each byte gets substituted with another byte (like a single S-box in DES); provides confusion and guarantees non-linearity of AES
- **ShiftRow**
the 16 bytes are permuted (like a P-box in DES but on bytes, not bits); provides diffusion
- **MixCol**
each column in the 4x4 matrix is linearly transformed; provides diffusion
- **AddRoundKey**
the state is XORed with a 128 bit round key

Slight deviations. Before the first round, AddRoundKey is applied with the 0th round key (which equals the AES key). Otherwise, our first step would be ByteSub, which wouldn't have any cryptographic effect since the plaintext bytes would just be changed in a fixed manner (no key involved yet).

Also, the last round has no MixCol layer. This has the effect that decryption can be made to look very much like encryption (see Section 5.3 in our book for the details).

5.4 ShiftRow

The layer **ShiftRow** permutes the 16 bytes $c_{0,0}, c_{1,0}, c_{2,0}, c_{3,0}, c_{0,1}, \dots, c_{3,3}$ as follows:

$$\begin{bmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,3} \\ c_{1,0} & c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,0} & c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,0} & c_{3,1} & c_{3,2} & c_{3,3} \end{bmatrix} \mapsto \begin{bmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,3} \\ c_{1,1} & c_{1,2} & c_{1,3} & c_{1,0} \\ c_{2,2} & c_{2,3} & c_{2,0} & c_{2,1} \\ c_{3,3} & c_{3,0} & c_{3,1} & c_{3,2} \end{bmatrix}$$

5.5 MixCol

Again, arrange the 16 bytes as a 4×4 matrix with entries in $\text{GF}(2^8)$. The **MixCol** layer transform this 4×4 matrix by multiplying it with another, fixed, 4×4 matrix:

$$\begin{bmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,3} \\ c_{1,0} & c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,0} & c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,0} & c_{3,1} & c_{3,2} & c_{3,3} \end{bmatrix} \mapsto \begin{bmatrix} x & x+1 & 1 & 1 \\ 1 & x & x+1 & 1 \\ 1 & 1 & x & x+1 \\ x+1 & 1 & 1 & x \end{bmatrix} \begin{bmatrix} c_{0,0} & c_{0,1} & c_{0,2} & c_{0,3} \\ c_{1,0} & c_{1,1} & c_{1,2} & c_{1,3} \\ c_{2,0} & c_{2,1} & c_{2,2} & c_{2,3} \\ c_{3,0} & c_{3,1} & c_{3,2} & c_{3,3} \end{bmatrix}$$

Example 140. For instance, the new byte at the position $c_{2,1}$ is

$$\begin{bmatrix} 1 & 1 & x & x+1 \end{bmatrix} \begin{bmatrix} c_{0,1} \\ c_{1,1} \\ c_{2,1} \\ c_{3,1} \end{bmatrix} = c_{0,1} + c_{1,1} + x c_{2,1} + (x+1)c_{3,1},$$

where all computations are to be done in $\text{GF}(2^8)$.

5.6 AddRoundKey

The **AddRoundKey** layer simply xors the current 128 bit state with a 128 bit round key.

The **key schedule** is as follows. Like for the states, arrange the original 16 byte AES key k in a 4×4 matrix with columns $W(0), W(1), W(2), W(3)$.

The i th round key is then obtained from the matrix with columns $W(4i), W(4i+1), W(4i+2), W(4i+3)$, where $W(4), W(5), \dots$ are recursively constructed:

$$W(i) = \begin{cases} W(i-4) + W(i-1), & \text{if } 4 \nmid i, \\ W(i-4) + \tilde{W}(i-1), & \text{if } 4 \mid i. \end{cases}$$

Here, $\tilde{W}(i-1)$ is obtained from $W(i-1)$ as follows:

$$W(i-1) = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ w_4 \end{bmatrix} \implies \tilde{W}(i-1) = \begin{bmatrix} S(w_1) + x^{(i-4)/4} \\ S(w_2) \\ S(w_3) \\ S(w_4) \end{bmatrix}.$$

Note that w_1, w_2, w_3, w_4 each are bytes. The function S is the ByteSub substitution. Since that substitution is nonlinear, the round keys are constructed from k in a nonlinear manner (unlike in DES).

As usual, the computation $S(w_1) + x^{(i-4)/4}$ happens in $\text{GF}(2^8)$.

5.7 ByteSub

The **ByteSub** layer takes each of the 16 bytes y and replaces it with the byte $S(y)$. As in DES, we could simply describe the (invertible) map by a lookup table. However, like the other steps of AES, it has a very simple mathematical description which we'll discuss next time.