Preparing for Midterm #1

MATH 126 — Calculus III
Friday, Feb 19

Please print your name:

Problem 1. Consider the points $P = (1, 2, -1)$, $Q = (2, 3, 3)$ and $R = (-1, 1, 2)$.

(a) Determine the vectors $\overrightarrow{PQ}$ and $\overrightarrow{QR}$.

(b) Find the distance between P and Q.

(c) Find a parametrization for the line through P and Q.

(d) Find a parametrization for the line segment from Q to R.

(e) Find an equation for the plane through P, Q and R.

(f) Find the distance between P and the line through Q and R.

(g) Find an equation for the sphere with center P and radius 5.

(h) Find the area of the triangle with vertices P, Q and R.

Problem 2. Consider the vectors $\boldsymbol{v}_1 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$, $\boldsymbol{v}_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ and $\boldsymbol{v}_3 = \begin{bmatrix} 3 \\ 1 \\ -1 \end{bmatrix}$.

(a) Determine $|\boldsymbol{v}_1|$.

(b) Determine $\boldsymbol{v}_1 \cdot \boldsymbol{v}_2$.

(c) Determine $\boldsymbol{v}_1 \times \boldsymbol{v}_2$.

(d) What is the angle between $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$? (Between $\boldsymbol{v}_2$ and $\boldsymbol{v}_3$?)

(e) Determine the projection of $\boldsymbol{v}_1$ onto $\boldsymbol{v}_2$. (The projection of $\boldsymbol{v}_2$ onto $\boldsymbol{v}_3$. Explain your answer!)

(f) Find a vector which has length 3 and is parallel to $\boldsymbol{v}_1$.

(g) Are there two of the three vectors which are parallel? Perpendicular?

(h) Find a parametrization for the line through $(1, -1, 2)$ which is parallel to $\boldsymbol{v}_1$.

(i) Find an equation for the plane through $(1, 2, 3)$ which is perpendicular to $\boldsymbol{v}_1$.

(j) Find an equation for the plane through $(1, 2, 3)$ which is parallel to both $\boldsymbol{v}_1$ and $\boldsymbol{v}_2$.

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Problem 3. Consider the plane described by $3x - y - 2z = 4$.

(a) Determine a normal vector for the plane.

(b) Find the x-intercept, the y-intercept and the z-intercept of the plane.

(c) Find a unit vector perpendicular to our plane.

(d) Find an equation for the plane through $(1, 2, 3)$ which is parallel to our plane.

(e) Is the line with parametrization $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1+t \\ 2+t \\ 3+t \end{bmatrix}$ parallel to our plane?

(f) If possible, find the intersection of the line with parametrization $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-2t \\ 3+t \\ 1 \end{bmatrix}$ and our plane.

(g) Determine the distance between the point $(1, 2, 3)$ and our plane.

(h) Find (a parametrization for) the line in which our plane intersects with the plane $x + y + z = 2$.

Problem 4. Consider the line L_1 parametrized by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2-t \\ 1+3t \\ t \end{bmatrix}$.

(a) Find a parametrization for the line parallel to L_1 through the point $(1, 2, 3)$.

(b) What is the distance between the point $(1, 2, 3)$ and the line L_1?

(c) Let L_2 be a second line, parametrized by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1+2t \\ -6t \\ 1-2t \end{bmatrix}$. Are L_1 and L_2 parallel? Are they the same line?

(d) Let L_3 be a third line, parametrized by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1+t \\ 2-2t \\ 5-t \end{bmatrix}$. Do L_1 and L_3 intersect? If yes, find their intersection.

(e) Let L_4 be a fourth line, parametrized by $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1+t \\ 8-2t \\ 3-t \end{bmatrix}$. Do L_1 and L_4 intersect? If yes, find their intersection.

Problem 5. Set up an integral (but do not evaluate) for the length of the curve $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-3t \\ \cos(2t) \\ t^2 \end{bmatrix}$ with $t \in [\pi, 2\pi]$.

Problem 6. Find (a parametrization of) the tangent line to the curve $P(t) = \begin{bmatrix} (t+1)\ln t \\ (2t+1)^2 \\ t^3 \end{bmatrix}$ at $t = 1$.

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