Example 14. In three-dimensional space, a point can be described by **cartesian coordinates** (x, y, z). See Section 11.1 in our book for the kind of pictures we drew in class.

- Mark the points $(0, 0, 0)$, $(x, 0, 0)$, $(0, y, 0)$ and $(0, 0, z)$ on the coordinate axes.
- Complete these points to a "box". The point furthest away from the origin is (x, y, z).
- Label the three remaining vertices of the box with their coordinates.

Example 15. We are interested in the distances between points.

- In 2D, recall (and justify) that the distance between (x, y) and $(0, 0)$ is $\sqrt{x^2 + y^2}$.

  The justification is Pythagoras theorem.

- In 3D, the distance between (x, y, z) and $(0, 0, 0)$ is likewise given by $\sqrt{x^2 + y^2 + z^2}$.

  We justified this using Pythagoras theorem twice. (You can find the argument in our book, too.)

The **distance** between the points $P_1 = (x_1, y_1, z_1)$ and $P_2 = (x_2, y_2, z_2)$ is

$$\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}.$$

Can you justify how this follows from the simpler formula we derived above?

Comment. Instead of $P_1 = (x_1, y_1, z_1)$, the book just writes $P_1(x_1, y_1, z_1)$.

Example 16. What is the distance between the points $(1, 2, 3)$ and $(-1, 0, 5)$?

Solution. The distance is $\sqrt{(-2)^2 + (-2)^2 + 2^2} = \sqrt{12}$.

Example 17. Interpret the following equations geometrically:

(a) $z = 0$

(b) $x = 0$, $z = 0$

(c) $x = 0$, $y = 2$, $z = 0$

(d) $z \geqslant 0$

(e) $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$

Solution.

(a) This is the xy-plane.

(b) This is the y-axis (a line).

(c) This is just the point $(0, 2, 0)$ (on the y-axis).

Comment. We are working in 3-dimensional space. By specifying 1 equation (here, $z = 0$) as constraint, the dimension is reduced to $3 - 1 = 2$. Likewise, by specifying 2 equations (here, $x = 0$, $z = 0$) as constraint, the dimension is reduced to $3 - 2 = 1$. Finally, by specifying 3 equations (here, $x = 0$, $y = 2$, $z = 0$) as constraint, the dimension is reduced to $3 - 3 = 0$ (a point is 0-dimensional).

In general, more complicated equations will result in more complicated shapes than planes and lines.

(d) This the half-space consisting of all points "above" the xy-plane.

(Above is in quotes, because it depends on our choice of having the z-axis point "up".)

(e) This is the first **octand**. (Why is there 8 of these?)

[Just like $x \geqslant 0$, $y \geqslant 0$ is the first quadrant.]

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