Preparing for the Final

MATH 227 — Calculus III
Monday, May 2

Please print your name:

Problem 1. Redo all practice problems for Midterm 1 and Midterm 2!

(The problems below will only cover the new material.)

Problem 2. Go through all the quizzes!

Problem 3. Consider the vector field $\boldsymbol{F} = \begin{bmatrix} x\sin(xy^2 - z) \\ \ln(x^2 - z^2) \\ xzy \end{bmatrix}$.

(a) Compute $\operatorname{div} \boldsymbol{F}$.

(b) Compute $\operatorname{curl} \boldsymbol{F}$.

(c) Which of the expressions $\operatorname{div}\operatorname{curl} \boldsymbol{F}$, $\operatorname{div}\operatorname{div} \boldsymbol{F}$, $\operatorname{curl}\operatorname{div} \boldsymbol{F}$, $\operatorname{curl}\operatorname{curl} \boldsymbol{F}$ are nonsense? Compute those that make sense. [Computational! Save second part for last.]

(d) Express the divergence and curl of a vector field $\boldsymbol{G}$ using the operator ∇.

Problem 4. Let C be the positively oriented boundary of the region defined by $x^2 + y^2 \leqslant 4$, $x + y \leqslant 2$. Spell out (i.e. express as ordinary integrals) the following line integrals:

(a) $\oint_C f(x, y)\mathrm{d}s$,

(b) $\oint_C f(x, y)\mathrm{d}x$,

(c) $\oint_C f(x, y)\mathrm{d}y$,

(d) $\oint_C \boldsymbol{F} \cdot \mathrm{d}\boldsymbol{r}$, with $\boldsymbol{F} = \begin{bmatrix} f(x, y) \\ g(x, y) \end{bmatrix}$.

Problem 5. Let C be the positively oriented boundary of the quadrilateral T with vertices $(0,0)$, $(1,2)$, $(1,3)$, $(0,4)$.

(a) Evaluate the line integral $\oint_C xy\mathrm{d}x - x^2\mathrm{d}y$ using Green's Theorem.

(b) Evaluate the line integral $\oint_C xy\mathrm{d}x - x^2\mathrm{d}y$ directly.

(c) Do you expect the value of the line integral to change if C was replaced by a different curve through the same four points?

(d) Evaluate the line integral $\oint_C 2xy\mathrm{d}x + x^2\mathrm{d}y$ in the most economical way.

(e) Do you expect the value of this second line integral to change if C was replaced by a different loop through the same four points?

(f) Evaluate the line integral $\int_L 2xy\mathrm{d}x + x^2\mathrm{d}y$ where L is the line segment from $(0,0)$ to $(1,2)$ followed by the line segment from $(1,2)$ to $(1,3)$.

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Problem 6.

(a) Is the vector field $\boldsymbol{F} = \begin{bmatrix} 2\cos(2x-y) \\ \cos(2x-y)-1 \end{bmatrix}$ conservative? If so, determine a potential function.

(b) Is the vector field $\boldsymbol{F} = \begin{bmatrix} 2\cos(2x-y) \\ -\cos(2x-y)-1 \end{bmatrix}$ conservative? If so, determine a potential function.

(c) Is the vector field $\boldsymbol{F} = \begin{bmatrix} 3x^2y^2z - z \\ 2x^3yz - 3y^2 + z \\ x^3y^2 - x + y + 2 \end{bmatrix}$ conservative? If so, determine a potential function.

Problem 7. Let C be the straight-line segment from $(0,3,-1)$ to $(4,-1,3)$. Let D be the curve parametrized by $\boldsymbol{r}(t) = t^2\boldsymbol{i} + (3-2t)\boldsymbol{j} + (2t-1)\boldsymbol{k}$, from $t=0$ to $t=2$.

(a) Evaluate the line integrals $\int_C x(z+1)^2 \mathrm{d}x + y\mathrm{d}z$ and $\int_D x(z+1)^2 \mathrm{d}x + y\mathrm{d}z$.

(b) Evaluate the line integrals $\int_C \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{r}$ and $\int_D \boldsymbol{F}\cdot\mathrm{d}\boldsymbol{r}$ with $\boldsymbol{F} = \begin{bmatrix} 3x^2y^2z - z \\ 2x^3yz - 3y^2 + z \\ x^3y^2 - x + y + 2 \end{bmatrix}$ (as in the previous problem).

(c) Write down an integral for the length of the curve D. No need to compute the integral; its numerical value is 7.19.

(d) Determine the average value of $f(x,y,z) = y + 3z$ on D.

Problem 8. Let R be the region defined by $2x + 2y + z \leqslant 6$, $x \geqslant 1$, $y \geqslant 1$, $z \geqslant 0$.

(a) Determine the volume of R.

(b) Determine the average value of $f(x,y,z) = x$ on R.

Problem 9. Let R be the region defined by $1 \leqslant x^2 + y^2 + z^2 \leqslant 4$, $x \geqslant 0$, $y \geqslant 0$, $z \geqslant 0$.

(a) Write down an integral for the volume of R using spherical coordinates. Then, compute it.

(b) Write down an integral for the volume of R using cylindrical coordinates. Then, compute it.
[The bounds are somewhat tricky!]

(c) Write down an integral for the volume of R using cartesian coordinates. [Similar comment.]

(d) Write down an (ordinary) integral for the average value of some function $f(x,y,z)$ on R.
[It is up to you to choose which coordinates you prefer to use.]

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