Quiz #9

MATH 126 — Calculus II
Friday, Nov 6

Please print your name:

Problem 1. Under which condition does $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converge?

Solution. The p-series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ converges if and only if $p > 1$. □

Problem 2. Determine whether the following series converge or diverge. Make sure to indicate a reason!

(a) $$\sum_{n=1}^{\infty} \frac{n-2}{n^3 - n^2 + 3}$$

(b) $$\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$$

(c) $$\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}\, 4^n}$$

Solution.

(a) $\sum_{n=1}^{\infty} \frac{n-2}{n^3 - n^2 + 3}$ converges by limit comparison with the converging series $\sum_{n=1}^{\infty} \frac{1}{n^2}$.

Indeed, let $a_n = \frac{n-2}{n^3 - n^2 + 3}$ and $b_n = \frac{1}{n^2}$, then $\lim_{n \to \infty} \frac{a_n}{b_n} = 1$ (do it!). Hence, $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges.

(b) $\sum_{n=1}^{\infty} \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$ diverges by limit comparison with the diverging series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}}$.

Indeed, let $a_n = \frac{1}{2\sqrt{n} + \sqrt[3]{n}}$ and $b_n = \frac{1}{\sqrt{n}}$, then $\lim_{n \to \infty} \frac{a_n}{b_n} = \frac{1}{2}$ (do it!). Hence, $\sum_{n=1}^{\infty} a_n$ converges if and only if $\sum_{n=1}^{\infty} b_n$ converges.

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(c) $\sum_{n=1}^{\infty} \frac{5^n}{\sqrt{n}\, 4^n}$ diverges because $\lim_{n \to \infty} \frac{5^n}{\sqrt{n}\, 4^n} = \infty \neq 0$. □

Problem 3. For which values of x does $\sum_{n=0}^{\infty} 2^n x^n$ converge? Evaluate the series (as a function of x) for these values.

Solution. $\sum_{n=0}^{\infty} 2^n x^n = \sum_{n=0}^{\infty} (2x)^n = \frac{1}{1-2x}$ provided that $|2x| < 1$ (or, equivalently, $|x| < 1/2$). □

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