Quiz #8

MATH 126 — Calculus II
Friday, Oct 30

Please print your name:

Problem 1. Determine whether the following series converge or diverge. If they converge, determine their value.

If a series diverges, make sure to indicate why!

(a) $\sum_{n=1}^{\infty} \frac{n}{n+1} =$

(b) $\sum_{n=1}^{\infty} 4^{-n} =$

(c) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}} =$

Solution.

(a) $\sum_{n=1}^{\infty} \frac{n}{n+1}$ diverges because $\frac{n}{n+1} \to 1 \neq 0$ as $n \to \infty$.

(b) $\sum_{n=1}^{\infty} 4^{-n} = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n - 1 = \frac{1}{1-\frac{1}{4}} - 1 = \frac{1}{3}$

(c) $\sum_{n=1}^{\infty} \frac{3}{\sqrt{n}}$ diverges because it is a p-series with $p = \frac{1}{2} \leqslant 1$. □

Problem 2. Use the integral test to determine whether $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$ converges or diverges.

Solution. $\sum_{n=1}^{\infty} \frac{n}{n^2+4}$ converges if and only if $\int_1^{\infty} \frac{x}{x^2+4} \mathrm{d}x$ converges.

First, however, we should verify that the integral test indeed applies: the function $\frac{x}{x^2+4}$ is obviously positive and continuous for $x \geqslant 1$. For $x > 2$, it is also decreasing, because $\frac{x^2+4}{x} = x + \frac{4}{x}$ is increasing (its derivative is $1 - \frac{4}{x^2}$, which is positive if $x > 2$).

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Substituting $u = x^2 + 4$, we find

$$\int_1^\infty \frac{x}{x^2+4}\,\mathrm{d}x = \frac{1}{2}\int_5^\infty \frac{\mathrm{d}u}{u} = [\ln|u|]_5^\infty = \infty$$

because $\lim_{u\to\infty} \ln|u| = \infty$. Hence $\int_1^\infty \frac{x}{x^2+4}\,\mathrm{d}x$ diverges, and we conclude that $\sum_{n=1}^\infty \frac{n}{n^2+4}$ diverges. □

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