

# Midterm #2

Please print your name:

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**Problem 1.** Using the integral test, determine whether the series  $\sum_{n=2}^{\infty} \frac{1}{n \log n}$  converges.

**Problem 2.** Determine the following limits.

(a)  $\lim_{n \rightarrow \infty} \frac{5^n + 3^n}{4^n - 1} =$

(b)  $\lim_{n \rightarrow \infty} \frac{7n^2 - 8n}{2n^2 + 3} =$

(c)  $\lim_{n \rightarrow \infty} \sqrt{\frac{3 + 2n^2}{1 + n + n^2}} =$

(d)  $\lim_{n \rightarrow \infty} \cos\left(\frac{n}{n^2 + 1}\right) =$

**Problem 3.** Write down the geometric series. Under which condition does it converge, and what does it converge to?

geometric series	=	provided that:
	what it converges to	condition for convergence

**Problem 4.** Under which condition does  $\sum_{n=1}^{\infty} \frac{1}{n^p}$  converge?

**Problem 5.** Determine whether the following series converge or diverge. Make sure to indicate a reason!

(a)

$$\sum_{n=2}^{\infty} \frac{1 - \log(n)}{1 + \log(n)}$$

series converges
 series diverges

Indicate a reason:

(b)

$$\sum_{n=1}^{\infty} \frac{n+1}{n^3+1}$$

series converges
 series diverges

Indicate a reason:

(c)

$$\sum_{n=2}^{\infty} \frac{7^n}{n^2 4^n}$$

series converges
 series diverges

Indicate a reason:

(d)

$$\sum_{n=2}^{\infty} \frac{n + \sqrt{n} + 7}{3n^2 + 1}$$

series converges
 series diverges

Indicate a reason:

**Problem 6.** Consider the power series  $\sum_{n=1}^{\infty} \frac{n}{5^n} (x+1)^n$

(a) Determine the radius of convergence  $R$ .

(b) Let  $f(x) = \sum_{n=1}^{\infty} \frac{n}{5^n} (x+1)^n$  for  $x$  such that  $|x+1| < R$ . Write down a series for  $f'(x)$ .

**Problem 7.** For which values of  $x$  does  $\sum_{n=1}^{\infty} \frac{x^n + 1}{2^n}$  converge? Evaluate the series (as a function of  $x$ ) for these values.

**Problem 8. (Bonus!)** What is the value of  $\sum_{n=1}^{\infty} \frac{1}{n^2}$ ? [We don't have the tools to evaluate this series, but you might remember from class.]