Example 123. Determine the following limits:

In each example, try to first "see" the limit! Then, apply some technique (like L'Hospital) to confirm.

- $\lim_{n\to\infty} \frac{3n^2+n-1}{n^2+1} =$
- $\lim_{n\to\infty} \frac{e^n}{n^2} =$
- $\lim_{n\to\infty} \sin(\pi n) =$
- $\lim_{n\to\infty} \sin(n) =$
- $\lim_{n\to\infty} \frac{\sin(n)}{n} =$

  Note that $-\frac{1}{n} < \frac{\sin(n)}{n} \leqslant \frac{1}{n}$. Since our sequence is squeezed between two sequences which approach 0, ...
- $\lim_{n\to\infty} \cos\left(\pi - \frac{1}{n^2}\right) =$

  $\pi - \frac{1}{n^2}$ approaches π as $n \to \infty$. Hence, ...
- $\lim_{n\to\infty} \sqrt[n]{\pi^2} =$
- $\lim_{n\to\infty} \sqrt[n]{n^2} =$

  Observe that $\sqrt[n]{n^2} = n^{2/n}$ is in the indeterminate form "∞^0". By taking the $\log$, we get $\log\left(\sqrt[n]{n^2}\right) = \frac{2\log(n)}{n}$ which is of the (indeterminate) form "$\frac{\infty}{\infty}$". Now, apply L'Hospital! (In the end, don't forget to undo the $\log$ to get $e^0 = 1$ as your final answer.)

Example 124. $\lim_{n\to\infty} x^n = \begin{cases} \infty, & \text{if } x > 1, \\ 1, & \text{if } x = 1, \\ 0, & \text{if } -1 < x < 1, \\ \text{does not exist}, & \text{if } x \leqslant -1. \end{cases}$

If you think of a representative case for each situation, then the previous (important!) example becomes very simple:

- $\lim_{n\to\infty} 2^n =$
- $\lim_{n\to\infty} 1^n =$
- $\lim_{n\to\infty} (1/2)^n =$
- $\lim_{n\to\infty} (-1/2)^n =$
- $\lim_{n\to\infty} (-1)^n =$
- $\lim_{n\to\infty} (-2)^n =$

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